

THE MONIST

THE LOGIC OF THE SCIENCES.¹

THE work of science is carried on by an increasing number of special sciences, developed in response to the necessity of increasing specialization and division of labor. This elaboration of science has been going on with startling rapidity in recent years. Lord Macaulay tells us that in his day the sum of human knowledge was of such small compass and was increasing so slowly that an active student might fairly comprehend all of its main facts and principles in a single lifetime. But this is far from true to-day. Not only can no one now keep up with the new learning, but no one can acquire in any detail a tithe of the existing learning. To take all of the different courses offered in any one year at any one of our great universities, such as Columbia or Chicago, would require the time of an average student for four or five hundred years. Our educational industry is thus busy multiplying thought-tools for all of our social occupations, and to secure effective cooperation between these occupations it is necessary for everybody to have some comprehension of the main problems of each, and of the means of solving these problems. Each of the different branches of education is thus attacking the social problem from a different point of view. Each abstracts, for the sake of intellectual convenience, certain aspects of the problem of developing the larger union; and as these branches become more efficient in producing truths useful in dealing with those aspects, they employ more and more the scientific method. If, therefore,

¹ A chapter of a forthcoming book on Social Reconstruction.

we are to understand the scientific method of attacking the social problem, we must inquire how the special sciences are related to each other in the use of this method.

Since the days of Auguste Comte, students have been clearly aware that certain grand divisions of science arrange themselves logically in a hierarchy according to their degree of abstractness and of consequent scientific development. Comte stated the series as follows: mathematics, astronomy, physics, chemistry, biology, psychology (transcendental biology), sociology—each of these terms, standing for a scientific point of view, or group of studies, becoming more numerous and more intensively specialized. As these sciences are only divisions for convenience in one logically continuous social function of systematic reflection, there are no, and can be no, definitely fixed boundaries between the sciences. Indeed, the old boundaries are continually being obliterated by new sciences, such as symbolic logic, mathematical astronomy, astrophysics, physical chemistry, chemical biology, biological psychology, and social psychology. There seems to be in particular no logical justification for placing astronomy as one of the grand divisions of general science, since it stands for no general conception different from those of mathematics, on the one hand, and physics, on the other. Logic also should be placed first as the science of intellectual method in all problems. As the science of thinking, logic sharpens the thought-tools — categories — with which all science must work. It sharpens them—makes them valid, as we shall see later—by means of the categorical experience of unity.

Each of these larger divisions of science, indeed, is distinguished by the fact that it constructs its system of conclusions or truths from the point of view of a single conception, which we find we have to use continually in interpreting our universe. These major conceptions are unity, magnitude, energy, matter, life, consciousness, and society;

and
the
istr
each
com
com
der
sion
emp
tion
sion
Thi
—t
less
the
aft
scie
own

indi
is c

in t

each
con
Th
mat
of
the
ness

Be
in t
tend

and the divisions of science which respectively elaborate their meaning for us are logic, mathematics, physics, chemistry, biology, psychology, and sociology. Of course, within each major conception are developed numerous subordinate conceptual distinctions: as, for example, in unity are the concepts of continuity, consistency, harmony, likeness, order, peace, and their opposites. Each of these grand divisions of science, in elaborating its own main conception, employs necessarily without much questioning the conceptions developed by the logically and "positively" prior divisions, but not so distinctly those of the later divisions.¹ This is what Comte meant by calling the series a hierarchy—that the sciences severally develop historically from the less scientific to the more scientific or "positive" form, in the order of their abstractness as indicated, those coming after being dependent upon those going before for the scientifically reliable truths with which to deal with their own less abstract, more complex problems.

This logical arrangement of the general sciences is indicated in the accompanying chart (No. 1). The chart is constructed to suggest, in brief and sketchy form:

1. The enumeration of the grand divisions of science in their logical sequence.
2. That this sequence is determined by the fact that each division interprets the universe in terms of a central conception, in the process defining the conception used. Thus logic may be defined as the science of unity, mathematics as the science of magnitude, physics as the science of energy, chemistry as the science of matter, biology as the science of life, psychology as the science of consciousness, and sociology as the science of society.
3. That in defining its own conception of the universe,

¹ An apparent exception to this rule is the case of physics and chemistry. Because of our modern tendency to define matter in terms of energy, i. e., in terms of mass and motion, sometimes stated as electrical activity, chemistry tends to become a branch of general physics.

I. CLASSIFICATION OF THE SCIENCES.

GRAND DIVISIONS	LOGIC	MATHEMATICS	PHYSICS	CHEMISTRY	BIOLOGY	PSYCHOLOGY	SOCIOLOGY
Main concepts being defined	Unity	Magnitude	Energy	Matter	Life	Consciousness	Society
Main concepts used as previously defined by science		Unity	Magnitude Unity	Energy Magnitude Unity	Matter Energy Magnitude Unity	Life Matter Energy Magnitude Unity	Consciousness Life Matter Energy Magnitude Unity
Space and time aspects of scientific method	Subject Predicate Copula	Problem Solving process	Statics Kinetics	Valence Reaction	Species Evolution	Elements Development	Organization Progress
Synoptic branches of the sciences	Philosophy Metaphysics Epistemology	Symbolical logic Theory of numbers, of space, of infinity, etc.	Mechanics Astronomy Geology Chemical dynamics, etc.	Theory of matter Theory of matter Liquids Solids, etc.	Botany Zoology Ornithology Ichthyology Paleontology etc.	General psychology Social psychology Experimental psychology, etc.	General sociology Geography Anthropology Archaeology Ethnology History, etc.
Specialized branches of the sciences	Formal logic Argumentation Logical critique of sciences sciences, etc.	Arithmetic Algebra Geometry Trigonometry Analysis Calculus Functions, etc.	Gravity Sound Heat Light Electro-magnetism, etc.	Inorganic Mineral acids Halogens Metals, etc. Organic Alkalies Hydrocarbons Aldehydes Alcohols Vegetable acids Esters, etc. Aromatics Carbohydrates Dyes, etc.	Bacteriology Anatomy Physiology Histology Neurology, etc.	Proteins (will, or motor reactions) Reflexes (intellect, or cognitive) Instincts Affective (feeling, or evaluating reactions), etc.	Hygiene Economics Communication Pedagogy Ethics Politics Recreation Statistics, etc.

NOTE: For other classifications see: Bacon (*Intellectual Globe*), Comte (*Positive Philosophy*), Spencer (*Classification of the Sciences*), Bain (*Logic*, pp. 627 ff) Pearson (*Grammar of Science*, pp. 534 ff), Thompson (*Introduction to Science*, pp. 31 ff), Plant (*History of the Classification of the Sciences*).

each science, with the exception of logic, employs as means the defined conceptions of the logically prior sciences. Logic, being the science of the very process of developing conceptions, must rely, for the testing of its own definitions, upon the immediate consciousness of unity as practical consistency in experience. This is the "logic" of experimentation. Of course, all sciences use experiment, but logic has to rely on it alone for the "given" means of verification.

4. That the elementary space and time aspects of experience appear in the method of all science as discriminations of structure and function, fact and change. Each science regards its subject-matter as a given system developing, and aims to define the development, i. e., to state the succession of forms the system assumes.

5. That the branches of each science develop from the synoptic to the specialized form, the former being the earlier effort to assemble data covering a wide range which is later more accurately classified and defined.

Let us now examine how each of the general sciences employs the historic method in defining the unity of experience from the point of view of its special conception.

LOGIC, we have said, studies the method of the formation and solution of all problems. It is just thinking become critical of its own method, so as to make the solving of the problem more swift and accurate. Let us emphasize again that logic is the science of unity, explaining what unity is and how it is maintained by thought as consistency, continuity, harmony in our experience. "That which, by the law of its being, intellect seeks above all and perpetually pursues and loves, is harmony. It is for a home and a dwelling with her that intellect creates a world; and its admonition is: Seek ye first the kingdom of harmony, and all these things shall be added unto you."²

² Keyser, *The Human Worth of Rigorous Thinking*, p. 23.

"*The problem of logic* has a more general and a more specific phase. In its generic form, it deals with this question: How does one type of functional situation and attitude in experience pass out of and into another; for example, the technological or utilitarian into the esthetic, the esthetic into the religious, the religious into the scientific, and this into the socio-ethical and so on? The more specific question is: How does the particular functional situation termed the reflective behave? How shall we describe it? What in detail are its diverse contemporaneous distinctions, or divisions of labor, its correspondent *statuses*? In what specific ways do these operate with reference to each other so as to effect the specific aim which is proposed by the needs of the affair?"³ With logic in the wider sense, as a critical theory of the relation of the social functions to each other, we shall have to do later when we discuss the question of scientific method in sociology. But at this point, where we are dealing with the method of the reflective function as such, in its systematic form as science, we are concerned with logic in its more specific form as the critique of science. The business of logic in this narrower sense, then, is to define for us the function of thinking itself, and indicate the conditions under which it fulfils its function.

Thinking arises as an effort of our activity to pass out of conflict and failure of function into union and achievement of function. In this process, logic discriminates two methods: deduction and induction. Deduction takes the familiar form of the syllogism, A is B, B is C: therefore A is C,—reasoning from the nature of a known whole system to the nature of its particular parts. There is, therefore, in deduction no real progress or growth of thought into a larger or previously unknown unity, no real transformation of an existing conflicting system into a new unified system, but only a clearer definition of particular

³ Dewey, *Essays in Experimental Logic*, pp. 97-98.

things in terms of their places or functions in a system already well known. In induction, however, we do make progress in the transformation of conflict into cooperation by getting beyond the old system altogether. In accomplishing this, induction takes the form of the inference, at first in the form of a question: Given A, is A C?—which being interpreted, means, Is this present, conflicting, unsatisfactory system, A, resolvable into the harmonious, satisfactory system which I guess is C? In both the deductive and inductive forms of judgment the copula, "is," signifies a mental action. In deduction the act of solving the problem has taken place in the past, and we are only reviewing it to make clear the steps taken or the factors of the solution previously reached. Deduction, therefore, serves as an instrument of analysis—it tells what particulars to look for and expect in a given situation: if we have the system A, we infer we have as factors that make it up the particulars B, M, X, Y, etc. Thus deduction, by itself, simply reaffirms, reasserts, the existing present system, without transforming it or getting us beyond it.⁴ But if this present system is felt to be conflicting and unsatisfactory, then induction is already at work in it, trying to formulate the new, more satisfactory system into which the old one must develop. In thinking we oscillate between deduction and induction; both are necessary; either one used alone or overemphasized leads to confusion and disintegration of experience.

Thus, deduction is analysis, whose function is to bring to attention the factors of the old system. Of itself, deduction does not develop a new system, it merely emphasizes the old, it does not employ experiment—for the relations

⁴ Cf. Baldwin: *Genetic Logic*, Vol. II, pp. 331-3332: "Inductive inference, therefore, is the process of the constitution of wholes of meaning in this or that control sphere; while deductive inference or implication is a movement of thought, whether one of relatively simple inspection or one of relatively detailed elucidation, within the whole already in some sense grounded or established."

of the system are all known; and for the same reason it does not develop new truth. Induction, on the other hand, is synthesis, whose function is to construct a new system, in part of factors of the old, and in part of new factors developed in experiment. Induction alone, however, can no more develop a new system than can deduction. The analysis of deduction is required to bring out both the factors of the old system that must be thrown out as no longer serviceable, no longer true, and also those that are still serviceable and true. Thus, the more deduction insists on the old system, the more induction insists on the new. (Where we have the dangerous extreme idea of the civil law as insisting on subservience of the individual to the present system without any change in it, there we are sure to have the opposite dangerously extreme idea of the entire abandonment of the existing law and system, in favor of an impossible, entirely new system.)

Logical Nature of a Law.—What thinking is always trying to do, as a means of keeping up the unity and growth of experience, is to define the relation of something concerning the thinker to the whole in which it is a member. General philosophy calls this action of thought "defining the relation of the particular to the universal." Whether it be the relation of the organ to the organism, of the word to the sentence, of the factor to the solution of the problem, of the habit to the character, or of the individual to society—in every case it is the definition of the relation of a member to the body whose unity and identity it cooperates with the other members to maintain and express. This is what science means by a law. Whether it be a so-called natural law of physical nature formulated in a laboratory, or a civil law of society (which, in a sense, is just as natural) formulated in a legislature, the law is in principle the same: a statement of the observed and tested relationship of a member to a body of members in which it

has its meaning and its existence. The law thus universally is a statement of a set of given conditions and a logical (ultimately practical) consequence, in the form, If A, then B. Assuming A represents the general system or body as given, and B the member, then the deductive form of the law would be as above: if A, then B, indicating that in a given system or body, A, a particular member, B, is operating at a particular point in a particular way. The inductive forms of the law would be the reverse: if B, then A, indicating that, given a member, B, as behaving in a particular way, the system, A, of which it is a member is inferred from its behavior. So astronomers, from observation of the sun and planets have inferred the nature of the solar system; and zoologists, reasoning from a few bones of an extinct or a new species of animal, have constructed with considerable accuracy the whole skeleton. Science thus enables us to predict, to anticipate, to construct as an ideal a system of relations in which we ourselves must function as factors in the future, and by this anticipation science enables us to reconstruct our own habits so as to function efficiently in the new system.

Transcendental Action of Judgment.—How do our minds achieve this anticipation? How do they transcend past experience in the construction of the new system? How do they proceed to set up this new system in ideal, so that, as a working hypothesis, it shall express the probable new relationship of the member to the system, and, upon test, turn out to be the new truth that guides our action in meeting some later situation? Every act of thought, every judgment however vague or abbreviated, every sentence, has, as before stated, a subject and a predicate. In the live judgment, where thought is actively engaged in trying to identify something, to place it in a system of relations, the subject is the accepted, unquestioned aspect of the situation or thing presented; the predicate is the un-

certain, questioned aspect, something tentatively affirmed but not immediately apparent. That is, the subject is the fact aspect and the predicate is the hypothesis aspect. In the beginning of the act of thinking, as, for example, in trying to identify an object approaching in the distance, the judgment takes the form: This is what? But even in the beginning, the subject is not merely "this" and the predicate is not merely "what," for the subject will have some immediately identified and accepted characteristics, such as color, size, and movement, and the predicate will consist of an immediately suggested hypothesis or series of hypotheses, such as man, horse, etc., because of the fact that every stimulus—every excitant attended to—means that a response is going on, that habitual, reflex actions are taking place through race instinct and other unreflective reactions derived from the past, representing organic functions maintaining their adjustment with the environment. There is, in other words, a functional adjustment already at work when reflection begins, and reflection arises because the adjustment has become insufficient to enable our functions to maintain the unity of our experience. As long as we are unable to identify the object, we feel impulses to move in contradictory ways, corresponding to the various hypotheses we entertain about it. Is it a friend expected?—is it a mad dog?—is it a creature we have never seen before, but like a horse? Each of these hypotheses brings to mind certain characteristics of color, size, movement, etc., which we compare with the like characteristics of the object in question. The predicate as hypothesis, therefore, is a system of relationships developed as a solution of past problems of adjustment and temporarily accepted as a truth unquestioned for the purpose of identifying the subject and securing adjustment in the present problem. If, by the experimental tests of observation, motions, calls, etc., we find that the features and behavior of the

object are largely identical with those of our hypothesis, then we are able readily to identify it, classify it, take a definite, unified attitude toward it. If, on the other hand, we have no classification, no truth, no system, within which the object will cooperate as a member, then we have to develop a new class, a new truth to explain it. We observe it and stimulate it more carefully to get a larger number of its reactions and characteristics, at the same time comparing them with those of the class or system in our experience most nearly like the object, until our own reactions to it, beginning as if it were in the familiar class, become definite and assured as new habits in a new system. Thus, some of the factors of the old system are carried over into the new, to serve as the bridge of truth between the two. If it were not for these old factors, which represent old habits in ourselves found to be still useful in the new situation, we could not deal with the new object at all, in fact we should be unconscious of it. But by our direct experimental dealing with the new object we have set up new habits which define the new factors of the new system.

This problem of the development of the subject-predicate distinction is perhaps the most profound and far-reaching question in philosophy; for it involves the whole question of the reality of the growth of the universe, and, in human consciousness, the question of how a new idea or thing can possibly originate. The critical cases in point are the earliest experiences of infancy, and the inventive activity of the genius—the scientific discoverer or artist. My own view is that consciousness, as human beings experience it in its reflective form, is not, in its elements, essentially different from the universal evolutionary process of adjustment by stimulus and response, but is just an advanced stage in that process. The critical case of rudimentary reflection in earliest infancy suggests that distinctions of subject and predicate must, in some sense,

antedate birth as factors of adaptation and of growth. If we are to hold consistently the doctrine of an evolving universe we must believe that consciousness is not an alien entity breaking into evolution from outside at some point—say, where self-maintenance or where self-consciousness is assumed to begin; but is rather, in its essential elements, the universal process of adaptation itself, existing not only in the sub-human and sub-animal worlds, but also in the so-called inanimate world, as the function of stimulus-response in all activity, now being recognized in the electrical theory of matter. In this stimulus-response process, all the way down the line the consciousness of the higher, later forms seems, as a matter of observed fact, to help in giving direction to the process in the lower, earlier forms—conspicuously, for example, in the case of parent and child—but not without the initiative and cooperative effort of the latter. In the case both of the child and of the inventor making a new discovery—wherever intelligence is promoting growth—it may well be that suggestions from higher intelligences are cooperating with the initiatives of the lower to produce the adaptations necessary for growth in both. This view seems to do justice to the claims of both the intuitionists and the rationalists, and offers a logical basis for a democratic theology, a democratic politics, and a democratic educational policy.

Practical Nature of the Criterion of Truth.—We have been maintaining that logic is the science which explains the meaning of thinking, in terms of the concept of unity or consistency. As already suggested, this conception means primarily the system of our own personal activities co-operating to maintain a satisfactory continuity of our experience. In the last analysis, it is the progressive realization of this conception in practice which serves us as the criterion of truth. As every problem means an effort to change a confusing, uncomfortable diversity into an in-

tel
of
the
the
giv
is
TH
dis
ing
the
tai
ge
inc
TH
for
coo
coo
tru
of
the
ph

wh
ical
ity-
sun
bey
kno
tion
giv
p. 2
I th
the
on
has
kno
per
Th
gui
act
clu
pro
...
as
ten

telligible and, somehow, comforting unity, any statement of conditions that guides us in making the transition, and thereby is identified as a consistently cooperating factor in the solution, we call truth; any statement which does not give us the solution, does not lead to satisfactory unity, or is inconsistent with it, we regard as error or falsehood. The phases of inference, deduction, induction, agreement, disagreement, etc., which logic discriminates in our thinking, are all subordinate functions in our effort to formulate the conditions of the situation, so we can make them maintain the growing continuity of our experience in the emergency when the rapid development of new, incompletely incorporated factors threatens to disorder and disrupt it. The necessity of maintaining the union requires that we formulate the law by which we can make the new factor cooperate to that end as true; or else, finding it cannot thus cooperate, that we cast it out as false. The standard of truth, as for all other values whatever, is the maintenance of the union. In other words, experience *as a whole* is the standard by which we test the truth or worth of any phase of experience.⁵

⁵ The process has been illustrated by the case of a man lost in the woods, who, trying to find his way home, has to devise a theorem, plan, or hypothetical map of his situation, in which the known features in his immediate vicinity—the slope of the land, the direction of the streams, the direction of the sun, etc.—are used as suggestions of the unknown parts connected with them beyond his observation. He proceeds to solve his problem by going from the known to the unknown in accordance with his hypothesis, and reaches a solution when he proves his hypothesis true by arriving home. This illustration, given originally by Professor Dewey (see his *Essays in Experimental Logic*, p. 237ff), has been criticized by Professor Baldwin, who dissents (incorrectly, I think) from this view of the criterion of truth, which he calls the "action theory." (See his *Genetic Logic*, Vol. II, p. 345ff.) The controversy turns on the old question of the nature of success. Professor Baldwin asks, "What has constituted the correctness, or truth, of his plan?... How does the man know his action is successful? The only answer is... by the *perceptual experiences* found to be what the thought or memory presented in image.... The test, then, is a *perceptual experience fulfilling the details of the plan that guided his action*.... The 'success' necessary, therefore, does not attach to acting thus or so." Professor Dewey answers (*Essays*, p. 240), "If we exclude acting upon the idea, no conceivable amount or kind of intellectualistic procedure can confirm or refute an idea, or throw any light upon its validity. If by acting in accordance with the experimental definition of facts, viz., as obstacles and conditions, and the experimental definition of the end or intent, viz., as plan and method of action, a harmonized situation effectually

Significance of This View of Truth and Its Criterion.—Now this doctrine—that the standard is the union, and that truth is instrumental, that truth is an invention which helps to maintain the union, and, when ceasing thus to function, must be repaired or discarded in favor of a more effective form,—this doctrine has some interesting logical implications.

In the first place, union or experience as a whole must mean more than what we ordinarily mean by individual experience. As the individual is a social member, experience as a whole must mean social experience so organized that, as it focuses in the individual, the individual's activities are satisfactorily unified and his services to the union increased in value. Likewise, the union must mean the universe so organized that the individuals carrying on its functions find their activities satisfactorily unified and effective for supplying their needs. (This can be made clearer when we come to discuss the relations of the social functions to each other.)

In the second place, this doctrine of the instrumental nature of truth and the normative nature of the common life, completely reverses the fundamental relations of these conceptions as they were held from the time of the ancient Greeks to our own day. Formerly, the universe being regarded, not as growing, but as fixed, truth was also regarded as a fixed thought-copy of the universe of which portions were progressively made known to the individual,

presents itself, we have the adequate and the only conceivable verification of the intellectual factors. If the action indicated be carried out and the disordered or disturbed situation persists, then we have not merely confuted the tentative positions of intelligence, but we have in the very process of acting introduced new data and eliminated some of the old ones, and thus afforded an opportunity for the resurvey of the facts and the revision of the plan of action." In other words, the test and criterion of truth is the practical unity of our experience as a whole resulting from our acting on an idea or plan assumed to be true. Being lost means disunion and confusion culminating perhaps in hunger and death. Home means not merely a "perceptual experience," but union, consistency in active experience—which is the test of success. Any other idea of the matter has across it the shadow of the old conception of thought as a mere copy of an ontologically external reality.

not mainly or in principle through the effort of human intelligence (which was considered as alien to the truth), but through revelation by "authorities" acting by "divine inspiration." This was the dominant theory of truth quite naturally during the centuries when, the means of knowledge being limited and monopolized by the few, the masses of the people were like children under the tutelage of their autocratic rulers. The masses have largely outgrown despotic tutelage. The democratic revolution has brought, as its very heart, a thought-revolution as sweeping as that of the Copernican theory of the solar system. It is therefore no accident, in this day when the citizen is becoming the sovereign, that truth as an autocrat is being replaced by truth as a servant. Man is not made for the law, but the law is made for man. If, then, man—mankind—organized, united, growing society in which the individual attains real freedom,—is indeed the end and standard of all validity and worth, we are under the imperative necessity of defining that standard more accurately than we have yet done.

In the third place, truth, as our most important means of keeping up the continuity and growth of experience, is, as has been suggested, a bridge between the past and the future. Like all other bridges, it should be built as strong and permanent as possible, and should be kept in good repair with a minimum disturbance to the traffic. Every emergency of experience requiring the action of judgment to maintain its continuity, is like a chasm over which we find we must pass, and truth is the tested, reliable bridge, binding the two sides together into one continuous experience. Every passage over the bridge of truth to some extent reconstructs it, enlarges its meaning and usefulness—but specialists, called scientists and philosophers, are required to assist in reconstructing it fast enough to sustain the increasing commerce of the world. Any inexperienced or careless tampering with the bridge is, of course, resented

by humanity as dangerous to the common welfare. The bridge derives its form, quality, and permanence from the nature of the traffic it serves, not *vice versa*. No doubt, this way of putting the matter is sufficiently irritating to the feeling of those who hold to the older theory of truth as an external copy of a fixed reality, but the logic of modern science and practical thinking seems to be dead against their view. The fact seems to be that science and other forms of experience are continually improving the bridge of truth so that subsequent heavier trains of experience can operate to supply the growing needs of the world.

MATHEMATICS, like every other science, has its origin and application in the concrete problems of our social life, whether they be those of navigation and surveying, of buying groceries at the store, or of simply trying to reach an object with the hand. From these concrete problems mathematics abstracts, that is, gives attention to only the quantitative relationships (as 2, triangle, etc.), assuming or ignoring all the other aspects.⁶ Thus simplified, the subject-matter is easier to think about in a clear and definite way. The science begins, both historically and logically, in the axioms, that are logical definitions of the mere quantitative aspects of our own sensations and motor activities, out of which all our notions of magnitude arise. It is especially by reflection on our own movements,—their repetition, change of direction, and duration, embraced within the continuity of our consciousness—that we construct our time and space axioms of mathematics, such as " $2 \times 2 = 4$ " and "The shortest distance between two points is a straight

⁶ Professor Keyser, in his interesting lecture on mathematics, takes issue with this position (*op. cit.*, pp. 276-279). He points out the wonderful expansion of mathematics in recent years, and shows how logic and mathematics have met in the borderland of symbolic logic, as developed by Leibniz and others—and is inclined to identify mathematics with all strictly valid thinking. I believe it is more useful to regard logic as the prolegomenon of mathematics, and define mathematics as the science of the unity of magnitudes.

line." We may say that the simplest axioms constitute our consciousness in its lowest terms, as vaguely defined in the early experience of the infant, or in the beginning of our attention to particular objects in any problem. They are the premises in concrete thinking just because they are the simplest statements of the relationships between our initial conscious impulses, prior to which we are not able to distinguish any antecedent conscious conditions. By means of these initial definitions mathematics solves its simplest problems of quantitative relations; and then using these solutions as truths for the solution of more complex problems; and these again for others, it progressively constructs a series of judgments, each depending for its validity upon a prior, and forming a branching "tree" of subjects, from simple arithmetic to the higher theories of variable functions. These subjects constitute an elaborate series of thought-tools, adapted to guiding judgment in a series of practical problems of increasing complexity, from measuring a piece of goods to the construction of a railroad bridge, or the location of a star.

How, then, does mathematics, as a typical science, solve a problem? Take, for example, geometry, and the simple problem of determining whether the theorem be true that the sum of any two sides of a triangle, ABC, is greater than the third side. Of course, this is no longer a real problem to us of mature years because we have so often proved it true by acting in accordance with it in moving or going from one place to another. But there was a time in infancy when for each of us it was a real problem; and the question is how our minds proceed to determine the truth. In the first place, we deductively discriminate familiar aspects of the strange situation, ABC, confronting us. If the situation were entirely familiar or had no familiar aspects we should not in either case, be conscious of it as a problem at all. The problem consists in determining the

relationships of the unfamiliar to the familiar so as to comprehend the situation as a whole. The familiar means an old way of thinking and acting, the unfamiliar means a modification of the old into a new way demanded by the new situation. Let us assume that the old habit of action has been direct, i. e., movement aiming to reach things by a straight course; therefore in practice we have been accustomed to the axiom—without definite formulation but operating in the form of habit as a truth—that a straight line is the shortest distance between two points. The straight lines would be the familiar aspects, then, of the new triangular figure, and any of them constituting a side, as AC, would be shorter than any other line, as ABC, passing through the same points. Applying the same reasoning to each side in turn, the theorem or hypothesis is established as a truth that can be similarly used to develop still other truths and guide activity. In this process both the old familiar figure, the line, and the new unfamiliar figure, the triangle, have acquired new meaning. The line, by becoming a member in a new system, functions as a new thing in our experience; and the triangle, as being composed of lines in a definite relationship to each other, becomes defined and available for use in reaching other conclusions. Practically, we may say we have thus learned that we can reach a point not only directly but also, with more time and effort, by a roundabout way, if we have the two ways definitely related to each other in the same system or map of conduct.

Now mathematics constructs systems of relations between mere quantities or abstract magnitudes, and tests them out as truths, in the same general way that the biologist does his systems of bones. The typical form in which the mathematical systems are stated is the equation. A mathematical equation is just a symbolic or shorthand description of a series of operations ending in an an-

swer—a confirmed hypothesis which states a satisfactorily unified experience. If we say $6 \times 5 = 30$ we mean (since multiplication is a quick way of performing the operation of addition), take six like systems, each containing five like things, and assemble them all together into one system—and you will have thirty like things in the new system. To the person who performs this operation for the first time, thirty is a genuinely new system of experience, reached by taking six of the old familiar systems of five units, and assembling them so that they lose their old identity, their old separate unity, and merge into the new unity of the system of 30. Thus, an equation as a statement of a method of solving a problem, is not merely an expression of static, quantitative identity of the terms of the two sides, but rather an expression of dynamic, qualitative change of an old experience into a new one, in time and space. The operative symbols, \times (times) and $=$ (equals), indicate the dynamic character of the equation, the changes in the process; the quantitative identity of the two sides indicates the continuity in the process—one continuous transformation of the old into the new. The asserted equality of value of the two sides means that the process of transforming the old system of experience into the new is equivalent to the new.⁷

The work of mathematics is to make accurate and reliable our calculations of merely quantitative relationships, but obviously it cannot be our sole guide in the business of life. The very fact of work means energy, about which

⁷ Stated in terms of moral experience, this means that the conduct which achieves the end has all the value of the end; or, in other words, the end or ideal is the guiding, evaluating factor in conduct—not something lying *beyond* it, and therefore unattainable. The end is the function. In a democratic, united, normal society, "Virtue is its own reward," art is its own reward, fellowship is its own reward, industry is its own reward. If the process leads to a valuable result, it has the value of the result—to whoever realizes the value. The value of an end, an hypothesis, an ideal, is the degree in which the process of realizing it introduces order, freedom, and satisfaction into experience. The social problem is just that of enabling all the workers to experience the full value of their work—in other words, to achieve a democracy, a union in which they shall all be members.

mathematics, as such, tells us nothing but the quantities of it; but we need to know also how it may be developed, changed from a kind useful for one purpose into a kind useful for another purpose. For an explanation of this we have to look to the science of physics.

PHYSICS, then, takes another step in developing an answer to our social problem by making clear to us the nature of energy as the power of doing work. It analyzes the different kinds of energy that we distinguish by our different senses, as mechanical energy, sound, heat, light, electricity and magnetism, gravitation, expansion of gases, and in doing so treats of matter only as a manifestation of energy, that is, as mass in motion. As an attempt to explain from the standpoint of energy how a universe is developed, physics is also historical in its method. It is a deliberate effort to formulate the method by which energy is transformed from its "natural" into its artificial manifestations, to give us the means of catching energy at its birth, and training it to the work of ordering the universe to serve the interests of man. In doing this physics has its genetic classifications of things as gases, liquids, and solids—as atoms, molecules, and masses—each with its specific motion as a function of its energy. Thus, in telling us what iron is, physics tells us the amount of heat required to change it from a solid into a liquid and a liquid into a gas, tells us what its specific gravity is, how well it conducts electricity—in short, how it behaves in furnishing energy for the world's work.

But magnitude and energy are obviously not the only general aspects of the universe that we have to take into account in dealing with it satisfactorily. Our senses of taste and smell especially indicate that things are made of quite radically different materials, and it is the science of chemistry which undertakes to carry us a step farther

in controlling our commerce with the universe, by showing how we may produce things by controlling the materials of which they are made.

CHEMISTRY, as a science of matter, undertakes to tell us the story of the composition of things out of their physical elements, and points out how we can get control of the elements of some things that we want to use, by decomposing other things that we do not want to use (except as decomposed). When, for example, chemistry tells what water is, it tells how water has been historically developed, how we may produce it by extracting hydrogen and oxygen from things constituted partly of them, and then recombining them intimately in the proportions of two volumes of hydrogen to one of oxygen. The formula of water, H_2O , is thus a shorthand symbol of a material process of genesis (we may even say, of Genesis) in which the relations of chemical combination of certain elements are indicated. In its definition of things as elements of matter undergoing decomposition and recombination, chemistry employs the general conceptions of magnitude and energy developed by mathematics and physics, and, of course, their classifications, but adds classifications of its own, in the historical form as elements, simple (inorganic) compounds, and complex (organic) compounds. This, science tells us, is the historic order: elements, non-living compounds, living compounds; and chemistry aspires to point out the methods of chemical reconstruction throughout the entire series.

But, after all, when we want to build a bridge of iron, drink water, or make seeds germinate, we have to know more about the universe than mathematics, physics, and chemistry assume to tell us. We have to know more about the behavior of things, for some things have a remarkable power of initiating and controlling changes, physical and

chemical, that are not accounted for by the formulas of these sciences.

BIOLOGY, as the science of life, approaches another step toward a solution of our social problem by defining exactly the conditions under which certain things we call living, initiate and control changes so as to perpetuate their own identity—that is, cause changes to take place as a periodically recurring cycle that maintains and develops the body through which they take place, against the forces that would otherwise disintegrate it. So biology adds a new general conception or class of relationships to those of the logically prior sciences by directing attention especially to the life process, and defining things, iron, water, seeds, etc., as factors in this process. And just as logic, mathematics, physics, and chemistry, each from the point of view of its special conception, classifies an historical continuity of living relationships, by classifying living forms in an evolutionary series from the lowest—i. e., earliest—plants to the highest—i. e., latest—animals. Biology thus endeavors to show how an individual maintains itself, how it produces other individuals, and how types of living individuals develop into succeeding types.

Take an illustration of thinking involved in biology, say, that of a naturalist, whose business is to define the terms and conditions of our reactions to our physical environment, so we may maintain the unity of our experience under new physical circumstances. Suppose he is walking in a field and sees a bird fly and alight at a short distance. Since he is looking for field birds, having in mind the familiar system of their characteristics, he quickly notices the markings as those of a pheasant. If nothing in the situation seems to contradict the conclusion, his judgment may end there as a rapid alternation of induction and deduction, chiefly the latter. As he is looking for something new,

his mind is at the start in the inductive attitude, but the first perceptions having so vividly suggested the pheasant that that conception dominates his thinking, he deductively enumerates the characteristics of the bird before him and passes on to some other judgment. But suppose something in the situation does seem to contradict his first conclusion. Suppose he notices the bird has, in addition to some of the familiar characteristics of the pheasant, also other markings and manners not of that class as he has known it. The appropriate reaction toward the creature is thus in doubt—what the present system of experience is, is uncertain—and this uncertainty is reflected in the transformation of the original tentative conclusion into a working hypothesis to be experimentally tested. Our scientist whistles to the bird and hears in answer a strange call. He approaches it and observes more closely its appearance. This may suggest some other hypothesis, e. g., that the bird belongs to some other class than that of pheasant, perhaps more exactly a grouse or large quail. Repeated comparison of the characteristics of the bird with those of known species may convince the naturalist that he has discovered a new species of pheasant—determining conditions of new forms of human activity. If the species proves to be specially good for food, it may be rapidly killed off, or may be protected and multiplied with consequent rapid alteration of the insect and plant life of its environment, i. e., with considerable transformation of the world in directions determined by the interests of man.

This is what any discovery means: a dynamic transformation, not merely a static revelation. And let it be noted the transformation takes place, not only in the conditions of objective experience, that is, in those habitual experiences where reflection is not prominently concerned, such as dining, reaping, admiring colors, etc., but the transformation occurs also in the conditions of subjective ex-

perience as a change in the very truth by which the transition from the old to the new was accomplished. There is no sense in which any *particular form* of truth exists prior to the discovery—the thinking process through which it is constituted as outcome of the effort to maintain the unity of experience in the emergency of a conflicting situation. If it was formerly true by tested and accepted definition that all pheasants had brilliant plumage, then it was not true that some pheasants have drab plumage until after some exigency of maintaining a unity of experience resulted in the modification of the genus "pheasant" to include the species with drab plumage. In this way every fact added to existing knowledge transforms preexisting fact and makes truth different from what it was before. In a growing universe there is no absolute ultimate fact. But biology does not ask especially what part consciousness plays in the life process. This explanation is left for psychology to define.

PSYCHOLOGY defines consciousness as an especially expeditious method of control in the life process. It is awareness of relationships and values, in a transition of activities from confusion to cooperation, from disunion to union, the awareness itself operating to facilitate the transition, harmonize the relationships, and increase the values of the activities.* Of course, psychology is as unable to give a complete definition of consciousness as logic, mathematics, physics, chemistry, and biology are to define completely—that is, practically—their own special conceptions: for each conception can be *practically* apprehended only in the light of all the others, including that of society. Each science uses, as clear and definite for *its own purpose*, the conceptions of the logically prior sciences, but in doing so, as we

* The modern definition of consciousness excludes the idea that it is an entity composed of a "soul substance." The psychical or subjective is the system of evaluating functions that maintains and develops, at the same time and in the same degree, our personal identity and the unity of society.

have seen, it modifies, enriches, the meaning of the conceptions used; and it assumes as a vague background of its thinking the conceptions of the logically subsequent sciences. Thus psychology may undertake to define consciousness in merely physical terms, as a change of environmental, overt forms of activity (matter in motion) into organic—neural—activity, such that the transformation issues again in environmental activities. But if we stop here we have totally failed to define consciousness. For the essence of consciousness consists in the values and change of values of activities, with reference to the maintenance of our own personal unity. The concept of the physical thus gets a new meaning by the necessary introduction of the concept of value, inherent in consciousness. And the concept of value, or of consciousness (as used by psychology) vaguely anticipates the concept of society (as developed by sociology). Value is developed in the performance of a social function. The individual is a social member. We cannot understand therefore what consciousness means (that is, as something leading to practical ends) until we understand more definitely about society as sociology interprets it; for all practice is social, i. e., involves social relationships intrinsically. It is indirectly for the valuation and control of these intrinsic, practical, social relationships—for the satisfaction of human interests, the discharge of human functions—that the concepts of all science are developed.

In interpreting the phenomena of consciousness, psychology, true to the method of science, classifies its subject-matter in a genetic series as successive phases of consciousness—successive forms of judgment of increasing comprehension, adapted to situations of increasing social complexity—from vague and fleeting sensation through perception and reasoning to the higher forms of the moral and esthetic judgment. By these classes of judgment,

psychology undertakes to define in terms of conscious processes the various mental attitudes, habits, and ways of acting that people are observed to assume. But, as we have seen, these judgments presuppose certain social situations, to which they are relevant as the agencies through which social conditions are evaluated and adjusted in the development of the larger life.

SOCIOLOGY, then, undertakes to construct the classifications of the social situations which call forth the characteristic reactions of the individual, as defined in psychology. In other words, sociology endeavors to explain society as a system of cooperative relationships between persons so that the individuals cooperating can adjust their conduct to the satisfaction of their needs. In this work, social psychology has recently been developed as a connecting link between the older psychology and sociology, to show the relationship of the individual, as psychologically described, to the world of people, as described by sociology. Mob psychology, pathological psychology (that of insanity, etc.), and transcendental psychology (that of mediumship, etc.) also have been recently developed to interpret social phenomena that psychology seems best able to explain. But in interpreting these and all other social phenomena, sociology is obliged to employ all of the major concepts defined by the logically prior sciences: consciousness, life, matter, energy, magnitude, and unity; and in so doing, deepens their meaning.

Chart II, Classification of the Social Sciences, is inserted here to suggest, in addition to the points of Chart I, how the particular social sciences are rapidly developing as reflective specialization upon the various problems of social practice. If, as we are maintaining, any science is merely a clarified development of common-sense thinking, then the logic of any science is an exposition of how

its method does interpret and clarify ordinary thinking. Suppose we take, as a single illustration, the logic of law-making as the basis of the growing science of legislation. While doubtless legislators, as a rule, have not been conspicuous for the use of scientific method, yet as above suggested, the operation of intelligence in the making of civil laws proceeds by the same universal process of thinking as does the formulation of a law of physical nature. And

II. CLASSIFICATION OF THE SOCIAL SCIENCES.

SYNOPTIC SCIENCES

General sociology
Anthropology
History

Geography
Ethnology
Archeology

Social origins
Social evolution
Municipal sociology
Rural sociology, etc.

SPECIALIZED SCIENCES

*Sciences of Maintenance**Hygiene :*

Eugenics
Dietetics
Therapeutics
Sanitation
Labor
Family
Consumption

2 Economics:

Commercial geog-
raphy
Extraction
Technology
Management
Finance
Transportation

*Sciences of Reflection**3 Information:*

Language
Investigation
Statistics
Accounting
Journalism
Bibliography

4 Pedagogy:

Primary
Secondary
Higher
Technical
Homemaking
Medicine
Business
Communication
Education
Ministry
Government
Recreation
Art

*Sciences of Control**Ethics:*

Conduct
Theology
Homiletics
Missions
Philanthropy

6 Politics:

Legislation
Law
War and defense
Penology
Administration

*Sciences of Recreation**7 Fellowship:*

Etiquette
Entertaining
Sports
Play
Dancing

8 Esthetics:

Dramatics
Music
Literature
Painting
Sculpture
Architecture
Landscape art
Craftsmanship

legislators are surely as much in need of science as are physicians and engineers. If we can make clear, then, what the universal process is in thinking as applied to lawmaking, we may be able to make legislation as scientific as biology or physics. We have seen that any truth is a statement of the relation of a particular to a universal, of a member to a system, determined as a solution of a past problem, and used without question as a means of reaching

a new solution—the method of organizing a new system. Any body of civil laws is a set of statements of the relations of individuals to society, determined as the solutions of past problems securing the effective cooperation of individuals in maintaining the social union. As society grows, as population increases and becomes more interdependent, we find that the union is continually tending to break down at certain points, for new methods of cooperation are necessary to maintain it; hence, new laws, as statements of the new methods, are continually necessary to enable people to understand how to cooperate.

The scientific legislator proceeds like any other scientist. He observes a breakdown of the union at a particular point, formulates an hypothesis of the kind of system which at that point would maintain the union, tests his hypothesis by public discussion and perhaps overt trial of unenacted custom; then states the social reaction against violation in the form of specific penalties, the reaction in favor of maintenance, as conducive to union, in the form of positive provisions for carrying it out. As old habits often appear under new conditions in disguised forms, we are not always able to perceive their consequences readily and judge them accurately, as in cases where we continue to punish a man for stealing a horse and praise a man for stealing a railroad. This is due mainly to the fact that the social union is always outgrowing our consciousness of its extent and cooperative requirements, as amply indicated in some quarters by insistence on mere nationalism as against internationalism. Internationalism is simply the inevitable, coming larger union, whose new laws of relationship we are endeavoring to state in the constitution of a league of nations.

J. C. BUSHNELL.

TOLEDO UNIVERSITY, TOLEDO, OHIO.

THE DEVELOPMENT OF THE NOTION OF CAUSE.

THE animal kingdom is very obviously divided into very different species. The difference to the eye between a lobster and an eagle is so great that it is difficult to realize they have anything in common. The pre-evolution notion that specific acts of creation were required to beget each animal species, well suits the extraordinary diversities that present the animal kingdom in the guise of a disparate collection of eccentric shapes and fashions. If we think of the more fantastic of living animals, such as the lizard, the serpent, the kangaroo, and the sea-lion; add to them the extraordinary variety of size, shape, and habit revealed in the modern reconstruction of prehistoric animals; and contrast the polymorphous, almost invisible, ameba of the stagnant pool with the huge elephant of the jungle, we realize very vividly how nature has concealed the continuous course of evolution beneath a bewildering array of diverse forms.

When evolution culminated in man, this distribution into widely differing biological species ceased. The different races of men represent simple variations of the same biological pattern. The minor variations within the human family, such as differences in color, stature, and hair, are but the faint echoes of those deep diversities that seem to divide the animal world into species with no visible points of contact. The men who lived more than 400,000 years before the Sphinx set her watch on the desert, seem to have been, when mere minor variations are counted out,

biologically identical with the man of to-day. Evolution exhausted her stock of biological diversities by her previous prodigality, and continued to shape man in the same biological mould.

But within the evolution from primitive man to the civilization of to-day there is a parallel to animal diversity. Biological species have been succeeded by *mental* species. There is a difference of mental habit between the man of the pliocene and the man of the modern city comparable with the contrast between the crustacean crab and the avian swallow. Mental diversity and biological diversity cannot be compared as two mountains can be compared in size—it is remotely metaphorical to compare the mental distance between a Kant and a primeval savage with the biological distinction between an ameba and a tiger. There is a parallel, however, as evident at it is wide. Widely diverse biological species mark stages in a continuous evolutionary course. The history of human thought reveals a continuous mental development connecting many diverse mental species.

There is the force of a revelation in passing from the perception of the remarkable diversities to their underlying connection in the process of evolution. To appreciate the full uniting strength of evolution, the first vision of the earth as the habitat of apparently disunited kinds is an indispensable preliminary. The realization that men do think differently, and often so differently that they seem to be as divided mentally as the sea-urchin and the rhinoceros are divided biologically, during the different stages of human thought, should precede the attempt to understand how the mind of the primitive has grown into the mind of the modern European.

Wundt remarks that there are two sets of data for deducing the habits and thoughts of primitive man—*under* and *on* the earth. The skeletons from prehistoric graves

or the implements buried in the earth give relatively little information of value for the knowledge of the primitive mind. The debris of primitive culture scattered *under* the earth's surface do, however, support, if they do not prove, the inference that primitive culture in general has been very stable, and confirm the conclusion that the primitive mind can be studied *on* the earth. Wundt means that primitive habits of thought are still preserved in the backward races of the world. The first men have left no trace either of themselves or of their thinking; but they have bequeathed, if not their actual beliefs, their general habit of belief to many yet undeveloped races. By investigating the beliefs of the earlier civilizations as they are recorded in myth, folk-lore, epic, sacred writings, or inscribed monuments, more particularly by the study of living backward peoples, it is possible to distinguish with some degree of adequacy the mental species that line the route of evolution. Two sharply defined types of primitive belief (selected from among many) serve to indicate the mental chasm between earlier and later thought. An appreciation of this chasm prepares the mind for a more vivid apprehension of the remarkable movement of thought that has bridged its opposite and seemingly remote sides.

A modern novel illustrates a curious persistence of one way of thinking favored by the primitive mind. Since it is obviously a relic of a past mental habit its incongruous appearance in a modern setting emphasizes the difference it really establishes between the primitive and ourselves. The author announces his program in a preface. He intends, he informs us, to *prove* that women can succeed as well as men in the professions. His thesis, that women can paint and write as ably as men, is probably quite unexceptionable—perhaps is a modern advance on the primitive incredulity. His *proof* is as primitive as his thesis is modern. He dresses his two heroines as men, he sends

them into the arena of life, one as a painter, the other as a journalist; and he makes them succeed by the simple process of describing their success. This curious notion, that to prove that a certain event can happen it need only be made to occur in a novel, is a striking reversion to the primitive custom of securing a result by imitating it. Perhaps this prejudice may survive even the reign of woman; perhaps in some future novel a woman may attempt to prove, by disguising her hero in feminine dress and making him a successful pleader, that man, if he be given a chance, can be as eloquent as his sisters.

Though no modern seriously believes that we can deduce real events from imaginary occurrences in novels, the primitive genuinely believes that events can be produced by imitating them. Practices based upon this belief are so diverse and so universal among primitive peoples, and they persist so obstinately as civilization advances, that the habit of thought they represent, however this may be finally interpreted, is obviously a characteristic part of the structure of the primitive mind. To kill my enemy, argues the primitive, I need only make an image of him in wood or wax or clay and pierce this image with my spear. No doubt, we simplify his thought by speaking simply of an endeavor to secure a result by imitating it. He will probably sing charms, or pray, or annoint his image with magically potent blood or fat; he may even lay a real ambush. Sometimes he seeks divine or spiritual aid to help on his effort. But the central core, imitating an event to make it happen, appears again and again in the records of savage, and even of relatively civilized, practice. Wax models of ships were found in an Egyptian temple. By floating these models in water and making them sink it was supposed that the enemy fleet could be destroyed. Magicians have swung themselves, in many lands, to make the corn grow. Australian savages imitate the life history of the witchetty

grub to promote the reproduction of this particular food. Pouring out water occurs in rituals for producing rain. The list of instances, if made complete, would stretch out, like the procession of kings in *Macbeth*, "until the crack of doom."

Dr. Frazer reserves a special term, "imitative magic," for the many and varied rites presenting the common feature of imitating an event to make it happen. Imitation alone is not a sufficient explanation of the mental habit expressed in imitative magic; but the prevalence of practices in which we can easily detect the common element of attempting to produce events by some performance that resembles them, points clearly to a primitive habit of thought that marks a well-defined mental species. Guy Fawkes is still burned in effigy; but no modern would seriously propose, though assisted by an elaborate ritual, to execute a murderer by hanging his proxy.

The primitive mind differs from the civilized in its opinion of the animal. Wundt attempts to divide the development of human thought into four roughly defined stages. In the second or totemic stage men have much concern for their relations with the brutes. They include them in their ancestry, and they include them as members of their tribes or tribal groups. In Australia the ancestor of a tribal group may be a kangaroo, or a snake. "Emu men" class themselves with emus, or emus with themselves. The animals have often the status of senior members of the tribal group in which they are included. Wundt contrasts the totemic estimate of the animal with the civilized estimate by describing it as the exact inverse. The totemist considers the animal his superior: civilized man thinks of it as vastly his inferior. Primitive man must distinguish between the animal and himself: kangaroos, snakes, and all the varied fauna of the earth strike his eye and affect his senses differently from his fellows. But he makes his

distinction very differently from his more civilized descendant. The animal may be his fellow-tribesman and may be the progenitor of his tribe. It may be more magically potent, wiser, and more deserving of reverence, than the oldest and wisest of his human kin. An animal may have founded the rites and ceremonies that his tribe performs to maintain its existence. A view of the animal so contrary to every modern idea, whatever be the ultimate causes on which it rests, denotes a difference of mental habit so pronounced that it establishes a veritable separate mental species. By his practice of imitative magic and by his conception of the animal, primitive man is as separated mentally from man to-day as the elephant is biologically separated from the crow.

Strategical position is as important in thought as in military art. Warfare strives both for strategical hold and for strategical vision—strategical hold to concentrate attack on weak or crucial points, strategical vision to observe in full the enemy dispositions. The strategic view-point has the important parallel in thought. To put the right question, to secure the conception that arranges the facts in a significant way, is a primary aim of all inquiry. The philosopher seeks a point of view that will present the whole universe in its most significant aspects; but in all inquiries the human mind must be content to procure as many illuminating view-points as possible.

One survey of the course of human thought on its way from primitive notions of the world to modern ideas is obtained from the concept of "transformation." Many notions converge on, or diverge from, the idea of transformation. This conception, therefore, gives an aeroplane view—displaying currents of thought as a country is spread out before the aviator. The fairy godmother transformed a pumpkin into a coach for Cinderella. The pumpkin had no qualifications for conversion into a coach: it was meta-

morphosed by fairy magic. For the primitive, transformation is metamorphosis—a conversion of one thing into another that does not depend on any connection between the two objects. The cause of the metamorphosis resides in a force purely external to the object transformed; and anything can, generally speaking, be metamorphosed into any other. Certain stones in the sacred territory of an Australian tribe are believed to metamorphose, at appropriate times and under appropriate conditions, into kangaroos. In the Greek myth, Deucalion and Pyrrha repopled the world after the flood by the metamorphosis into human beings of stones they threw behind them. Metamorphosis, the transformation of unconnected things into one another, appears persistently in the myths, legends, and folk-tales of the world. The Prince is changed into a beast; Daphne becomes a laurel-tree; Zeus seeks Europa as a bull; Niobe is transformed into a fountain. Metamorphosis between animals and men remains persistently in lower cultures, as the most plausible residue of universal metamorphosis. The werewolf legend, of humans oscillating between the forms of wolves and human beings, is scattered through the world. Witches regularly become cats or hares in folk-tales.

The constant recurrence of metamorphosis in the stories of the world intimates a primitive belief in the possibility of transforming anything into virtually anything else. The metamorphosis is usually associated with some magic force that constrains the transformation. Novels written in the nineteenth century, though they contain fictitious incidents, reveal the writer's estimates of possibility. One author may introduce events regarded by his contemporaries as purely mythical, to constitute an allegory or to amuse. Fairy-tales are still written for children. Swift used the improbable to give edge to the satire of Gulliver. But the literature of any century, taken as a whole, reveals that century's estimate of possibilities. It is possible that the

future literature of the twentieth century may reveal clearly that both authors and public believe that the dead may return, that seers can read the future, and even that the sleeper can visit the country he will enter when he dies. If these beliefs become real and widespread estimates of possibility, they will become ordinary items of literature. Metamorphosis is an ordinary item in the early stories of the world, and thus represents a primitive estimate of possibility. The present beliefs of peoples in primitive and lower-grade culture confirm the evidence of literature and tradition. The concepts connected with transformation thus divide the primitive and modern minds into different mental species, just as do the concepts attached to imitative magic and to the relation between man and beast. The study of the concept of transformation provides a particularly favorable opportunity for observing how the beliefs of the primitive develop into the beliefs of modern man.

The human mind begins by supposing that any transformation is possible. As thought advances, the notion of development is substituted for the belief in metamorphosis. Transformation is seen to follow certain paths, and change is seen to depend on the qualifications of the objects that change. Looking down on the course of human thought from the standpoint of the idea of transformation, the concept of arbitrary metamorphosis, under the complete constraint of magical forces external to itself, gradually passes into the realization that things develop from, and into, one another under the regulation of laws. The belief in metamorphosis is replaced, gradually and surely, by the recognition that transformation proceeds along definite routes.

It has been often supposed that we attain to the notion of *cause* by observing series of events that conform to a regular sequence. Thirst disappears after drinking and, since drinking must precede the disappearance of thirst, we think of the drinking as cause and the disappearance

of thirst as effect. Hume thought that causes and their effects appear to us to be linked by a necessity because we notice that they constantly occur in succession. When B happens after A has happened and B never takes place without the previous occurrence of A, the belief naturally arises, Hume argued, that A produces B—that there is a necessary causal connection between A and B. The primitive conception of transformation as metamorphosis constrained by a magical force recognizes, however, a *tie* between cause and effect without recognizing the necessity, or even the existence, of uniform sequence. The notion of a tie between causes and their effects is the essential element in the concept of causality. A uniform sequence, as of day and night, may be repeated *ad nauseam* without coming under the adjective "causal." Wolves, tigers, and other beasts roamed through the gardens of Circe's palace. She transformed, by her magic arts, the companions of Odysseus into swine, as she had transformed other unfortunates into other animals. If she had metamorphosed each individual into a different beast she would have exerted causal actions that presented no uniformity of sequence. Or, if the metamorphosis man-into-beast were the sequence repeated, she might have transformed one man, and one man only, into a pig or tiger or other animal; there would still have been causal action though without any repetition of sequences. Uniform sequences need not be causal and causal connections need not be repeated—it is possible to think of repetitions as uncaused and it is quite possible to recognize that one event may cause another on its one and only occurrence in the universe.

One trend of modern thought threatens a curious fate for Hume's argument. Hume argued that uniformity or regularity of succession imposes upon us the notion of necessary connection or causation. Many thinkers now suppose that science is relieved by the existence of uni-

formities from the necessity of believing in causes. Science, according to their view, has merely to describe the uniformities that occur and formulate their rules. The universe has certain regular habits. Hume's argument that we deduce from regularity to cause is replaced by the notion that regularity only requires a formula or merely a description of the regularities observed.

The gradual recognition that transformations or events proceeded along definite routes did not create the concept of causation. The recognition of causal paths is a modification of the idea of cause and not its originator. The substitution for the idea of the metamorphosis of the seed into the plant of the notion that the one develops according to definite laws into the other, presented the original causal notion in a different aspect. Magical forces, exercised by a powerful magician or operating through prescribed ceremonies, compelled, in the belief of the totemist, the growth of the plant. The causal compulsion is simply differently conceived when it is realized that the qualities of the seed and the favoring influences of soil, warmth, moisture, air, and light conjoin to produce growth. This alteration of conception is one of the most important, if not the most important, truths imposed upon the human mind by experience.

Men must have realized from the first that appropriate means must be used to attain certain ends. The conception of a primitive belief in absolute magical forces that could virtually create any transformation whatever is therefore a limiting conception. Lotze remarks that causes were first appreciated in the workshop. The early stone-workers must have learned the technique of knapping flints. There would be no fire without rubbing together two sticks or twirling a hard point on a soft block or striking a flint. But the primitive mind was dominated by the notion of absolute magical forces that constrained changes and

events. Never or seldom, Dr. Frazer points out, do primitive peoples ascribe death to natural causes. The man does not die because a spear pierced his heart but because the thrower or his spear was magically potent. This domination by the magical contrasts sharply with the modern realization that the universe is a complex of causal paths. Slowly and laboriously the causal sequences that determine all events were brought to light. The conviction that the regularity of law ruled over all was gradually established.

The whole complex of causal paths is not yet unraveled. Perhaps it never will be; perhaps the belief in an absolute uniformity is as too absolute in one direction as the primitive belief in absolute magical forces was too absolute in another. Absolute uniformity, rigid determination, is, indeed, perhaps as much a limiting conception when applied to the modern mind as the notion of absolute magical forces is a limiting concept when applied to the primitive consciousness. In neither case does the principle hold *in toto*. The primitive combines some appreciation of defined causal sequences with his dominant notion of unrestricted magical forces (unrestricted except by their own degree of potency), and the modern mind is loth to believe that in the sphere of will, and it may be in nature, there is an absolute reign of law. But two dominant conceptions concerning all transformations and happenings separate the primitive from the modern mind. The primitive mind is dominated by the notion of metamorphosis—by the belief in the operation of magical forces that are unrestricted by causal paths. Magical potency converts life into death, seed into plant, or draws the fire from the stick. Experience slowly but surely replaces the principle of metamorphosis by the principle of orderly passage or development, as causal sequences come to light and force on the mind the recognition that causal action has sequences prescribed for it.

GLASGOW, SCOTLAND.

JOSHUA C. GREGORY.

THE SIX ENDINGS OF MARK

IN LATER MANUSCRIPTS AND CATHOLIC AND PROTESTANT
IMPRINTS OF THE OLD ARMENIAN VERSION.

SAMUEL Prideaux Tregelles, the Cornish Quaker, in the first scientific text of Mark ever published in Britain (Plymouth, 1857), says this in Latin and small print at Mark xvi. 8:

"After verse 9, the subscription,

According to Mark,

is read in the second column of the page in the Vatican Manuscript (the third column being left vacant), and on the next page the Gospel of Luke begins. In *ancient* manuscripts of the Armenian Version, the subscription,

Gospel according to Mark,

is read *here*; and then they either omit the remaining part of this Gospel or have it noted with a new title: *Gospel according to Mark.*"

Tregelles obtained these facts from Charles Rieu's collation of the Armenian Version printed at Venice in 1805. This edition and a reprint of it in 1860 (reported by Caspar René Gregory) are the only accurate ones known. Twenty New Testament manuscripts were used, and various readings were given in the margin by the editor, Zohrabian. When the Armenians first printed their noble, neglected old Version at Amsterdam in 1666, the Trinitarian interpolation in 1 John v. 7, the Mark Appendix and other additions which that Version had always protested against,

were added by the editor. Since then the printing of the Version has been chiefly in the hands of the Roman Church and the London and New York Bible Societies. On an island in the Venetian lagoon, the Mechitarist Fathers, in communion with Rome, print the Version of their ancient National Church in conformity with the Vulgate. It is true that many characteristic readings are allowed to stand, such as the Doxology in the Lord's Prayer (omitted by the Vulgate) and the primitive

When they heard

at Mark xvi. 8. But in such blocks of matter as the Mark Appendix and the Adultery Section in John, the Version has been conformed to the Vulgate. The New York Bible Society prints a similar distorted edition at Constantinople, though it omits 1 John v. 7, and puts warning notes at Mark xvi. 9 and John viii. 1.

We proceed to give the seven different ways of ending Mark which this unfortunate Version presents. "The *Seven Endings of Mark*" we might have called this article. But this would have been unjust to "the Queen of Versions": there is only one ending. The other six have nothing to do with the Armenian translators of the fourth and fifth centuries, but are later corruptions and compromises to please the Churches of Europe. However, we will number the endings as seven, beginning with the true one. Here they are:

1. The Primitive Ending, in all manuscripts before 989, and in careful ones down to the fifteenth century.

2. The Arision Ending, in one Ararat manuscript of 989 (often wrongly given as 986), where the Longer Appendix first appears, with the true name of the author: "Presbyter Ariston" (or Arision).

3. The Compromise Ending, in many manuscripts after the first Crusade, recognizing the unauthentic nature of

the Appendix by putting the red colophon ahead of it, and writing the Appendix in smaller letters and paler ink. This is the treatment in the Philadelphia Manuscript of 1116, owned by John P. Bedrosian.

4. The Double Ascription to Mark.

5. The Ascension-tide Lection.

6. The Bible Society's Ending (with a note about the absence of the Appendix in Greek manuscripts, but suppression of the fact that Armenian ones are the star witnesses against it).

7. The Roman Catholic Ending (same as the Bible Society's, but with no note at all). This ending is printed by the Armenian monastery aforesaid under the authority of the Pope. Only later manuscripts end thus.

We now proceed to print these actual Armenian endings, true or false, as translated by me from the originals with the assistance of Frank Normart and Shadrach Adurian, M.A., Anatolia College, Marsovan.

1. *End of Mark* in Armenian manuscripts before 989, and in careful ones down to the fifteenth century.

233 And when they heard, they went out and fled
I from the sepulcher, because they were terrified;
and they said nothing to any one, for they feared.

Gospel according to Mark.

(Rubricated colophon, having the force of: *Here endeth* etc.)

NOTE.—This is the only genuine Old Armenian ending, and it has never yet been printed in the original, and only in English in *The Monist* for 1917 and elsewhere in recent years. It is found in the Philadelphia Gospel Manuscript of 1098, owned by John P. Bedrosian (Peters) and in all known manuscripts before 989. *This means that it took the tenacious Armenians more than a thousand years to*

corrupt the Holy Gospel, but the Greeks only three hundred. For the Washington and London Manuscripts, fifth century, are the first to contain the Appendix, while the Armenians were still hesitating to copy it in the fifteenth! (Gospels in one volume, second century; Armenian Version, fourth and fifth.)

2. *End of Mark* in the Etchmiadzin Manuscript of 989.

- 233 And when they heard, they went out and fled from the sepulcher, because they were terrified; and they said nothing to any one, for they feared.—*Presbyter Ariston's*.

Now, when he was risen early on the first day of the week, he appeared first to Mary Magdalene [etc., as in our current versions. The words, *Presbyter Ariston's*, are crowded in as an afterthought].

3. *End of Mark* in the Peters Manuscript at Philadelphia, A.D. 1116.

(No era given. Should the Armenian Era of 552 be meant, the date would be 1667; but the manuscript appears to be much older.)

- 233 And when they heard, they went out and fled
I from the sepulcher, because they were terrified; and they said nothing to any one, for they feared.

Gospel according to Mark.

(Rubricated)

Now, when Jesus was risen early on the first day of the week, he appeared first to Mary Magdalene [etc., as in our current versions].

NOTE.—Conybeare's *Catalogue of Armenian Manuscripts in the British Museum* (1913) shows that many manuscripts end like this after the twelfth century, when the Crusaders began to influence the Armenian Church.

The smaller print above represents the smaller letters and paler ink of the Manuscript (which I have by me). Moreover, the rubricated colophon to the Appendix reads:

Introduction to the Gospel of Luke.

So determined were the first Armenian scribes who added the Appendix not to admit it as Mark's.

4. (Same as No. 3, except that the colophon is repeated as a new title for the Appendix. This arrangement is found in Zohrabian's printed edition of 1805.)

5. *The Ascension-tide Lection.*

- 233 And when they heard, they went out and fled
II from the sepulcher, because they were terrified; and they said nothing to any one, for they feared.

Read for the Ascension.

Now, when Jesus was risen early on the first day of the week [etc., as before].

NOTE.—This reading is noted by Zohrabian as that of the main copy used by him. It is found also in No. 21 at the British Museum (sixteenth century) and in another late manuscript owned by John F. Lewis, of Philadelphia.

6. *End of Mark* as printed by the New York Bible Society at Constantinople (editions of 1895, 1896, and 1900).

And when they heard, they went out and fled from the sepulcher, because they were terrified; and they said nothing to any one, for they feared.

*Now, when Jesus was risen early on the first day of the week [etc., as before].

*From here to the end of the chapter is missing, according to some, in Greek manuscripts. [Note by the Bible Society. It ought to add: *and in far more Armenian ones.*]

7. *The Roman Catholic Ending.*

And when they heard, they went out and fled from the sepulcher, because they were terrified; and they said nothing to any one, for they feared. Now, when Jesus was risen early on the first day of the week, he appeared first to Mary Magdalene [etc., substantially as in the Vulgate and the King James Version].

NOTE.—The transition to the Appendix without even a fresh paragraph is taken from a Mechitarist edition at Overbrook Seminary (Venice, 1740). My own Mechitarist edition (Venice, 1868) begins a new paragraph, but has no note.

It is high time that the Armenians were allowed to print their noble old Version from their own manuscripts, without our Church or presbytery standing over them.

CORRECTIONS

In former articles dealing with the Armenian Version, I relied on a free translation made by my friend Frank Normart. This led to several mistakes, as in *The Monist* for April, 1917, p. 170; October, 1918, p. 529. Also in my *Studies in the Christian Religion* (Philadelphia, 1915-1919). In the present article my friend's translation has been checked by my growing knowledge of Armenian and also with his concurrence.

ALBERT J. EDMUNDS.

PHILADELPHIA, PA.

PARACELSUS AS A REFORMER IN MEDICINE.

AS the main interest of Paracelsus (1493-1541) lay in medicine, and as he rejected the ancient authorities on the theory and practice of medicine, it was of first importance to his mission that he should formulate a theory of medicine that should harmonize with his philosophy of nature and the results of his experience and observation. Naturally also his medical theory is closely related to his natural philosophy.

The history of medical science gives ample evidence of a great need of radical reform both in theory and practice at the period of the activity of Paracelsus.

The accepted body of medical doctrines as authorized by the medical faculties and taught in the universities was founded upon the ancient authorities of Hippocrates and Galen and their Arabian interpreters, and particularly of the latter. The Greek physician Galen had indeed accomplished much in his time to advance the practice of medicine, and had even performed dissections, not indeed on the human subject, but upon animal bodies. But to the physicians of the time of Paracelsus the ancient Greek of Galen was almost unknown in its purity—but only as transmitted, commentated, and interpolated through Arabian interpreters—Avicenna, Averrhoës, Mesue, and others. The Galenism of the sixteenth century was a corrupted Galenism, overlaid with Oriental occultism and mysticism. Moreover, the medieval spirit still ruled in the profession.

The teachings of the Arabian-Greek authorities had been for centuries and were still held as infallible dogmas. The doctrines of medical science were a finished book, just as the authorities of the Church were final,—they might be commented, expounded, interpreted, and taught, but not contradicted nor seriously questioned. No experiments were encouraged, no doctrines or opinions tolerated that might be in evident contradiction to these sacred authorities. Though new diseases had arisen to puzzle the profession, no new unauthorized measures could be attempted to meet them. Naturally enough, while such a condition prevailed the medical profession was bound to degenerate into a self-satisfied caste. Naturally also ignorance and incapacity, fostered by the lifeless teaching of the conventional dogmas, theories, and the stereotyped system of symptoms and remedies, often gave rise to pretentiousness and hypocrisy. It followed also that in the Renaissance, when men were thinking many new thoughts—that there should have arisen a suspicion as to the sufficiency of the medical theory and practice, not perhaps within the ranks of the conventionally trained profession itself—though here and there a voice was raised in protest against some phase or feature of medical practice or theory—but more particularly among laymen, and the general public.

It was indeed during the very time when Paracelsus was acquiring his medical training, that Erasmus in his *Praise of Folly*, satirizing the follies of the time, said of the contemporary medical science, "And indeed the whole art as it is now practised is but one incorporated compound of craft and imposture." Agrippa von Nettesheim, the elder contemporary in Germany of Paracelsus, had also written,¹ "The greatest reputation is attained by those physicians who are recommended by splendid costumes, many rings and jewels, a distant fatherland, tedious travels, a

¹ Baas, *Geschichtliche Entwicklung des ärztlichen Standes*, p. 185.

strange religion, especially the Hindu or Mohammedan, and who combine with these a monstrous shamelessness in the praising of their medicines and cures. They observe times and hours most exactly, dispense their medicines always according to the astrological calendar, and hang all kinds of amulets on the patient. Simple and native medicines are quite neglected. Costly foreign remedies are preferred, which latter are mixed in such enormous number that the action of one is counteracted by that of another, so that no human sagacity can foresee the effects which will arise from such an abominable mixture."

Peter Ramus, the distinguished French humanist and reform professor in the College of France—himself a great admirer of the work and skill of Paracelsus, as shown in an essay urging certain reformatations in the University of Paris (in 1562)—emphasized the laziness of the professors of medicine and theology, and complained that the analyzing of herbs and simples and the study of their effects upon the body were totally neglected.² The shortcomings of the medical profession were evidently not unappreciated by many able contemporary critics.

The medical theory of the period was based, as already mentioned, upon the doctrines of Hippocrates and Galen. With these Greek physicians, medicine had been indeed a living science, though primitive. They at least had learned by observation and experiment; but their medieval interpreters no longer experimented, and their observations were only such as might enable them to apply the accepted doctrines and formulas of the ancient authorities. The teaching of medicine in the universities at the time of Paracelsus was practically confined to the reading of Avicenna, Mesue, Avernhoës, and other interpreters of the Galenic doctrine, and commentaries and exposition of their mean-

² Cf. Graves, *Peter Ramus and the Educational Reformation of the Sixteenth Century*, Macmillan, 1912.

ing by the lecturer. Dissections and laboratory methods were lacking; though sometimes at rare intervals, when permitted by the civil and clerical authorities, demonstrations in anatomy—superficial and crude indeed—were made in the presence of the medical students and the physicians of the town. The first important publication on anatomy, marking the beginnings of the development of modern anatomical studies, was that of Vesalius which appeared in print two years after the death of Paracelsus.

The authoritative theory of diseases was based upon the Galenic doctrine of the four humors or fluids of the body, phlegm, blood, the yellow and the black bile,—these being related by metaphysical analogy to the four elementary qualities—cold, dry, warm, moist. Any disturbances in the proper proportions of these fluids produced illnesses or disease. The nature of these disturbances was indicated by accepted symptoms. The treatment was directed toward restoring the supposed disturbed balance of qualities as indicated by the symptoms, and consisted generally of bleeding, purging, and the use of decoctions of herbs, generally extremely complex in their admixture. But through Oriental influences this Galenic theory, fantastic and unscientific as it was, had become complicated with astrology and other mysticisms, while the superstitions of the medieval Church, and the heathen superstitions of the northern European peoples, were not without their influence upon local medical practice.

Troels-Lund³ has interestingly described the prevalent beliefs of the sixteenth century as to the causes and cures of disease. They may be briefly summarized as follows:

First: Disease comes from God by his direct volition as warning or as punishment. The logical conclusion was that God should be permitted to effect the cure. Prayers, penances, and the offices of the Church were thus the nat-

³ Troels-Lund, *Gesundheit und Krankheit in der Anschauung aller Zeiten*.

ural instrumentalities through which the divine mercy might be invoked to relieve the suffering. Manifestly the skill of the physician had here little place.

Second: Disease comes from the influence of the Devil and his agents. Here again prayer, penances, exorcisms, and purification by the offices of the Church might avail (white magic). So also, however, might magic ceremonies and formulas, and exorcisms by wise women, and magicians, who presumably owed their power to their superior knowledge of the occult powers of nature, or perchance even to unholy alliances with the powers of evil (black magic). Here also there was little room for the skill of the physician, though it might be he could assist;—who could be certain?

Third: Disease comes from the stars. Here evidently the physician might help, who knew the secrets of the heavens, and who gathered and prepared his remedies at the auspicious time and could administer them when the planets were favorable.

Fourth: Disease comes from the disturbances in the fluids or humors of the body. This was the Galenic doctrine above mentioned.

A fifth general idea as to the cause of disease mentioned by Troels-Lund, may have been but an elaboration of the fourth, viz., that disease was owing to something lacking in the body which medicine could supply to restore as it were the equilibrium, and with this idea there was developed a body of *materia medica* during the sixteenth century which presented an astonishing catalog of often almost incredible and repulsive remedies.

To this question as to the causes of disease Paracelsus, in his desire to replace the ancient authorities by something more in accordance with his own philosophy of nature, applied himself with characteristic originality, and with some intuitive insight.

He catalogs and describes five "entities" which influence the health of man. These entities or influences are the *ens astrale*, or sidereal influence; the *ens veneni*, or influence of poisons; the *ens naturale*, or influence which exists in the nature of the individual, the microcosm; the *ens spirituale*, influences acting not directly upon the body but through the spirit (*Geist*); the *ens deale*—the will of God acting directly to produce illness by way of warning or punishment.

With respect to the first of these, the *ens astrale*, he recognizes the influence of the stars without admitting their control of the destinies of man, and he makes a curious attempt to explain their influence by the hypothesis of a kind of effluvia conveyed from the stars through the atmosphere.

His treatment of the second influence, the *ens veneni*, is of interest as illustrating both his comprehension of an important physiological fact, and his fanciful and imaginative elaboration of it into theory. "The body was given us without poison, and there is no poison in it; but that which we must give the body for its food contains poison."⁴ He elaborates this idea by explaining that the plant and animal food which we eat contain both useful and useless material, wholesome and unwholesome, food and poison. In the body the food and the poison must be separated, the food being transformed into flesh and blood and bone, etc., the poisons eliminated. This separation, he considers, is effected by the "Archæus," a directing force or spirit. The Archæus, situated in the stomach, sorts out and separates the wholesome from the unwholesome in the food. So long as the Archæus performs his functions properly our food is wholesome and the body thrives. Should from any cause the Archæus become ill or incapacitated the separation

⁴ Paracelsus, *op. fol.* (Strassburg, 1616-18), I, 9, "Paramirum."—Quotations in this article are translations by the writer from the original (German) texts.

is incomplete, and we suffer from the poisons being imperfectly eliminated. The Archæus is, then, says Paracelsus, an alchemist, for his functions are similar to those of the chemist in his laboratory. Other animals have their "Archæi," and their functions vary in degree from those of man.

"The peacock eats snakes, lizards, stellions; these are animals which in themselves are perfect and healthy, though to the needs of other animals sheer poison, but not to the peacock. For from whatever reasons it may be, his alchemist is so subtle that the alchemist of no other animal equals him, who so cleverly separates the poison from the good, in that which the peacock eats without injury.

"Observe, then, that every animal has food adapted to it and which has been ordained for it by its alchemist who separates the proper materials. To the ostrich there is given an alchemist who separates iron," etc.⁵

In everything there is an essence and a poison; an essence is that which preserves man, a poison that which produces illness.

It is difficult to say to what extent Paracelsus believed that this presiding Archæus was a true spirit having an individuality or personality of its own, to what extent a term to typify a force or principle. It is interesting to note that in the Latin text of his work *De gradibus* (1526), published by Huser after the manuscript of the pupil and amanuensis of Paracelsus, Oporinus, the following definition appears: "Archæus est ista vis quae produxit res, id est dispensator et compositor omnium rerum." The word *vis*, or "force," is here noteworthy, though not necessarily a demonstration of the exact notion possessed by Paracelsus himself.

To the philosophy of the neo-Platonists of the sixteenth

⁵ *Op. fol.*, I, 10, "Paramirum." This is an ancient fable that the ostrich can eat iron.

century, however, the notions of force and principle and spirit were more closely connected, for as God was the soul of the universe, and as man—the microcosm—possesses a soul, so also all other parts of the macrocosm had souls or spirits. Or, as expressed by Agrippa von Nettesheim,⁶ "It would be absurd if the heavens, the stars, and the elements, which are for all beings the sources of life and soul, should themselves lack these,—if every plant and every tree had part in a nobler destiny than the stars and the elements which are their natural begetters."

The description of the third influence, the *ens naturale*, or the influences dependent upon the nature of the individual, is more complicated. For man, the microcosm, was the epitome of the macrocosm, and in his nature were to be found in a sense the counterparts of all external influences. As in the external universe, the sun, moon, and planets have their predestined and determined courses, so the microcosm has its sun, moon, and planets with their predestined courses. As the heavenly bodies could exert some influence on the health and diseases of men, so the corresponding planets of the human organism may have similar influences. Thus, as the sun by its light and heat influences all living things, so the heart, the sun of the body, has its determined course and gives light and warmth to the body. To the moon and its influences corresponds the brain in man; similarly, the lungs correspond to Mercury, the liver to Jupiter, the kidneys to Venus, the gall to Mars, etc. Thus the planets have their analogies in the body, and each has its established course and influence, its conjunctions and appositions. These courses are, according to Paracelsus, foreordained at birth, and the time is set for their life and activities, as an hourglass is set for a determined time. "For example, a child is born at a certain hour, and is to live according to his *ens naturale* for ten

⁶ As cited by Cassirer, *Das Erkenntnisproblem*, I, 193.

hours, as had been predestined at its creation. Then the courses of its bodily planets will be completed just as if it had lived a hundred years. And the hundred-year man has no different course than the one-hour child, but a slower one. Thus are we to understand what the creation and predestination are in the *ens naturale*. Observe, however, that the other *entia* often interrupt the predestination."⁷

All this is fanciful and fantastic enough. The one fundamental observation underlying the elaborate metaphysical structure seems to be the recognition of the varying endowments of vital energy with which different individuals are provided at birth, and of the fact that not alone upon external influences is the health or illness of individuals dependent.

The fourth influence, the *ens spirituale*, is also treated in quite a fantastic manner as judged from our present point of view, though to a period when witches and sorcerers were tortured and burned, there was probably little in the thought of Paracelsus which might not be plausible enough to his contemporaries.

The *ens spirituale* comprises those influences which affect the body only indirectly by direct action upon the spirit (*Geist*). Paracelsus distinguishes between spirit (*Geist*) and soul (*Seele*).

"Take note that there is not comprehended in this *ens spirituale* any devil nor his effects nor his assistance (*Zulendung*), for the devil is no spirit (*Geist*): an angel also is not a spirit. That is a spirit which is born from our thoughts, without matter, in the living body: that which is born after our death, that is the soul (*Seele*)."⁸

The spirit may suffer from diseases like the body, but it must not be forgotten that when the spirit suffers the body suffers also.

⁷ *Op. fol.*, I, 14, "Paramirum."

⁸ *Ibid.*, I, 17.

He explains how these spirits may be created by the will of man when he thinks of another person, in waking or in sleeping hours—and the spirits thus engendered may attack the spirits of the person thus selected, and do injury to them and through them to their possessor. On the other hand, the spirit thus assailed may successfully resist and prevail over the attacking spirit, in which case the originator himself may be the sufferer.

In the discussion of this topic Paracelsus manifestly realizes that he is liable to come into dangerous conflict with the Church doctrines, if misunderstood, and warns his readers that they "lay aside the style which is called theological. For not everything is sacred which is called theological, and not everything is holy which theology employs. Also all is not true which he uses who does not understand theology aright. Though the theologians describe this *ens* most forcefully, though not under the name and text of our fourth 'Pagoyum' (?), they also deny what we maintain."⁹

This curious attempt to explain the mechanism of the then generally credited occult influence of one person upon another by magic or charms or witchcraft or the evil eye seems strangely foreign to our modern thought, but it is well to remember that such representative thinkers of that time and of later times—as Trithemius, Pico della Mirandola, Agrippa, Melanchthon, Cardanus, and Giordano Bruno, were all believers and writers or lecturers upon magical influences.

In the *ens deale* Paracelsus recognizes the influence of the will of God upon the health of men, but instead of accepting the inference that through the offices of the Church is help alone to be obtained, he emphasizes the idea that God has created the system of nature and that He prefers to work through nature rather than by direct inter-

⁹ *Ibid.*

ference. The true physician, therefore, is he who understands the phenomena of nature, and is through that knowledge the agent through whom God acts. This point of view is a dominating thought with Paracelsus and is brought forward continually in many of his works. As God may send illness so he sends the physician at the proper time when the period of punishment is completed, for naturally only then may the cure be effected.

"When He performs a miracle, He performs it humanly and through mankind; if He effects wonderful cures, He does that through men, and therefore the physician."¹⁰

He admits that there may be two kinds of physicians, those that heal through the faith, and those who heal through their skill in medicine. Not all have sufficiently strong faith, but the end of the period of punishment having arrived, the physician may cure through the art of medicine. Curing by the power of the Christian faith, he explains, moreover cannot apply to the heathen—Turks, Saracens, Jews, etc., but asserts that he teaches the foundations of medicine not only for Christians but for all others as well.

"The physician is the servant of Nature, and God is the master of Nature."¹¹

"But that you may know what the reasons are that God has created medicine and the physician because He is the physician, and yet works through the physician and does not Himself act without a physician, understand this explanation, that such is His mystery that He does not will that the sick shall know that God is the physician, but that the art may have a procedure and a practice, and that man shall not perceive His help in miracles alone, that is, in God Himself, but also in His creatures that

¹⁰ *Ibid.*, I, 21c.

¹¹ *Ibid.*, I, 22.

they may help through the artist in medicine, and that according to His predestination in its proper time."¹²

"So know then all, that we human beings are born naked and bare, and bring with us neither knowledge nor wisdom, but await the grace of God whatever He may send us. And He gives us nothing as a free gift but life. Whether we be well or sick, that He commands through nature; teaching us to speak, that He ordains through our parents; and so on as we grow up, we must learn all things with labor and difficulty, for we possess not the least knowledge. As, then, we must learn, there must be something which is not human that teaches us. For man at first can do nothing. If we then wish to learn, our first foundation is in God, that we acknowledge Him as our God who teaches us and sends us what is needful. And if we consider all things well we find that all things take place through an instrumentality which God has provided at the creation. Thus God the Father, when He created the heavens and the earth, created them to be an instrumentality through which that should come to us over which our bodies should rule. Thus is man the master of medicine, of the fields, the meadows, and the vineyards."¹³

This formal cataloging and characterization of the five *entia* which influence the health of man, by no means adequately present the whole theory of disease entertained by Paracelsus. He also characterizes disease itself as an organism. Troels-Lund well summarizes his theory as follows: "It is not, as the Arabians accepted, something only negative in relation to positive health. It is itself something positive. It is a form of life of its own, a parasite organism, a microcosm. Man is in illness of two natures, has at the same time two bodies in one and the same. To understand this rightly we must make it clear

¹² *Ibid.*, I, 22-23.

¹³ *Ibid.*, I, 113.

what that is we call life. Life is always an intimate union of three constituents: salt, sulphur, mercury. So long as life lasts they form an intimate union and are not noticed. But if they begin to separate and to become separately noticeable in pains and burnings, this is disease and it may lead to complete separation: to death. Life is something invisible while its elements are kept together. If life ceases they separate and become visible. You do not understand this? Try it. A tree lives. Cut it into firewood and it dies. When you now burn it, that which burns is sulphur, that which vaporizes is mercury, and that which is ashes is salt. There is nothing more in it. All these three, the combustible, the volatile, the insoluble, are found united in everything living and are separated only when it dies. These three it is which we characterize by the names of sulphur, mercury, salt."¹⁴

"Disease is a conflict between two invisible forms of life—disease and health, which are both harbored in the same organism. The conflict is carried on everywhere in the body; is felt as heat, cold, discomfort, pain in all regions. The fever, the pain, are not the disease but only expressions of the force, the form, under which the nature of the organism, the inner alchemist or archæus, or whatever you choose to call the living force within you, seeks to put to flight the disease. The main battle consists in the crisis. If the 'archæus' wins, the disease must dissolve—give way, and be expelled as perspiration, excreta, respiration. If the disease conquers, the organism is dissolved in death."¹⁵

"Disease itself he viewed as a half spiritual, half corporeal living organism, as a microcosm within the microcosm, as a kind of parasite—with its own life phenomena and life processes within the human organism; its healing takes place when nature or medical art succeeds in devel-

¹⁴ Troels-Lund, *op. cit.*, pp. 156-7.

¹⁵ *Ibid.*, p. 159.

oping so forceful a vital activity that the parasite is suffocated, that is, the disease is overcome."¹⁶

Another and more modern phase of thought which is much emphasized by Paracelsus is the curative power which lies in nature herself, independent of all medical assistance.

"That you may understand what it is that heals wounds, for without that knowledge you may not readily recognize the remedy, you must know that the nature of the flesh, of the body, the veins, the bones, has in it an innate balsam (*mumia*) which heals wounds, thrusts, and such like things. That is to say, the balsam lying in the bone heals the fracture, the balsam naturally contained in the flesh heals the flesh. So with every member, it must be understood, each has its healing in itself and thus nature has in every member that which heals the wounded part. Therefore the surgeon should know that it is not he that heals, but the balsam in the body. If the physician thinks it is he that heals he deceives himself and does not understand his art. But that you may know for what purpose you, the surgeon, exist, learn that it is to provide a shield and protection to nature in the injured part against enemies, so that these external foes may not retard, poison, nor spoil the balsam of nature, but that it may remain in its balsamic power and influence by the maintenance of such protection. Therefore he who can protect and take good care of the wounds is a good surgeon."¹⁷

"In nature's battle against disease the physician is but the helper, who furnishes nature with weapons, the apothecary is but the smith who forges them. The business of the physician is therefore to give to nature what she needs for her battle. . . . Nature is the physician."¹⁸

¹⁶ R. Julius Hartmann, *Theophrast von Hohenheim*, 1904, p. 90.

¹⁷ Paracelsus, *Chir. Bücher* (1618), "Grosse Wundartzney," p. 2.

¹⁸ *Ibid.*, p. 207.

These medical theories of Paracelsus were extremely heretical in the eyes of the medical profession of the time. It was not possible for him to have publicly maintained his theories without exciting the opposition of the medical faculties and practitioners. Least of all was that possible in the universities which were the very strongholds of conservatism.

The practice of his profession differed as decidedly as did his theories from the conventional methods of diagnosis and treatment. Having broken with the teachings of the ancient authorities, the young physician had not hesitated to learn from all sources which were open to him in his travels in his own and in foreign lands and his sojourning among all classes of people, the remedies and treatments used by all kinds of healers and the homely remedies in use among the common people. His chemical knowledge and his chemical theories of the nature of vegetable or mineral substances in their relation to the nature of man doubtless suggested new ideas, and these he tested by observation and experience. To what extent these new methods were original with him, and to what extent accumulated during his wanderings in foreign lands or among the villages of Germany or Switzerland, it is not possible to state. Certain it is that many of the remedies and treatments he used and taught were new to the medical literature of his time. The complex syrups and decoctions of rare and costly herbs he rejected, and taught instead that the true aim of chemist and physician was to separate from medicinal raw materials their effective principle, spirits, or *arcana* by the application of chemical processes. In this line of work he set the example of using, instead of the complicated and irrational mixtures of the medieval pharmacopœia, simpler extracts and purer medicinal preparations, both mineral and vegetable.

Whatever be the final judgment as to the relative im-

portance of Paracelsus in the up-building of medical science and practice, it must be recognized that he entered upon his mission with the zeal and the self-assurance of one who believes himself inspired with a great truth, and destined to effect a great advance in the science and practice of medicine. By nature he was a keen and open-minded observer of whatever came under his observation, though probably also not a very critical analyst of the observed phenomena. He was evidently an unusually self-reliant and independent thinker, though the degree of originality in his thought may be a matter of legitimate differences of opinion. Certainly once having from whatever combination of influences made up his mind to reject the sacredness of the authority of Aristotle, Galen, and Avicenna, and having found what to his mind was a satisfactory substitute for the ancient dogmas in his own modification of the neo-Platonic philosophy, he did not hesitate to burn his ships behind him, and cut loose from the dominant Galenism of his time, and to preach and teach that the basis of the medical science of the future should be the study of nature, observation of the patient, experiment, and experience, and not the infallible dogmas of authors long dead.

As we follow the story of the lifelong struggle of Paracelsus against the centuries-old conservatism opposed to him, it is impossible not to feel great sympathy not only for the cause for which he labored but also for the self-sacrificing devotion and tremendous earnestness which he brought to his work.

We can realize now at this distance that the condition of medical science and teaching was in his day at a very low ebb. Improvement was indeed hopeless so long as dogmas held as infallible inhibited all initiative toward rational criticism or new experiment. We can see that the insistence of Paracelsus upon the study of the patients and their diseases rather than of ancient books, his emphasis

upon the value of experiments, upon the application of chemistry to the understanding of physiology and pharmacology, his own radical innovations in the use of new and unauthorized remedies, and his denunciations of the hollowness of much of the medical practice and teaching of his time—that these were all working in the direction of progress.

Realizing this we can make allowance for his crudities, his limited understanding of the goal toward which his labors tended, his superstitions, his pseudo-science. We can sympathize with this lonely figure battling throughout his life to break the chains which held medical science enslaved, seeing the path which must be followed to build that science upon surer foundations—yet himself too much hampered by the medieval point of view, too little versed in the methods of modern science to clearly lead the way toward the goal he struggled to attain.

But though we recognize the importance of the work of Paracelsus, while we admire the earnestness and essential sincerity of his reform campaign, we should be unfair to his opponents of the conservative school of medicine, if we failed to recognize the shortcomings of Paracelsus which were in part responsible for the lack of appreciation and of following which he could command during his life. Modern historians of medicine, while recognizing the importance and the essential sincerity of the work of Paracelsus, have not been blind to these shortcomings.

Dr. Jos. Bauer¹⁰ summarizes the reform influence of Paracelsus thus:

“In order to infuse new life into the sluggish and torpid mass of science, there was needed a giant spirit, who with strong hand, regardless of authority and dogma, should seize the reins, and undisturbed by the judgment of his time should understand how to sweep away the accumu-

¹⁰ *Geschichte der Aderlässe*, 1870, p. 146.

lated dross. All these qualities the reformer Hohenheim possessed in the highest degree, and he ennobled these gifts by an unselfish honest spirit though his inclination to extravagances drove him into a fanaticism which amounted to a complete autocracy in the domain of opinions. In order to maintain these he trod underfoot the bounds of propriety and in that way alienated the sympathy of calm thinkers."

The medical system of Paracelsus was not adapted, in Dr. Bauer's opinion, to influence the physicians of his time, and his ideas were carried forward by a relatively small number of followers—often visionaries, and whose extravagances often did much to discredit his thought. So also Häser,²⁰ while acknowledging the great value of the services of Paracelsus to medicine, the purity of his enthusiasm and his earnestness, nevertheless recognizes that the methods he used to attain his aims in the science were mistaken.

"This contempt for the foundation of scientific medicine," says Häser, referring to Paracelsus's sweeping rejection of the importance of anatomy as a foundation of medicine, "is in all times the symbol of all transcendental as well as of all empirical systems. . . . With Paracelsus this undervaluation goes so far that he only uses the word 'anatomy' to denote that which in his opinion should form the foundation of medicine, the knowledge of the nature of life."²¹ "Above all he manifests the strong love of freedom native to the German and Swiss stock. 'No one can be another's who can be his own.' This native self-consciousness was as with Luther, with whom he had much in common that is good, and with John Brown, with whom he had much in common that is bad, nourished by the fact that he was lowly in origin, was born and lived in poverty,

²⁰ *Lehrbuch der Geschichte der Medizin*, Jena, 1882.

²¹ *Op. cit.*, p. 91.

and that a rude bringing-up separated him from the finer manners of the cultivated classes. The neglect and slight which he experienced insulted his pride and drove him back into himself. By blameworthy or unblameworthy misfortunes he arrived at that arrogant disdain so peculiar to strong but unbending natures, through premeditated contempt for the great accomplishments of his contemporaries to overestimate his own power and his own accomplishments."²²

One of the later writers upon the place of Paracelsus in the history of medicine, Dr. Hugo Magnus,²³ after commenting upon the condition of medical science of the time for which the dictum attributed to Rhazes might well have served as a motto, "The study of a thousand books is more important for the physician than seeing a thousand patients," says, "That our hero soon felt the lamentable condition of his science gives very certain evidence of a sound and lively critical sense in matters medical. And that he soon gave expression to this dissatisfaction in powerful attacks upon the corrupt conditions must ensure him at all events our sympathy. This fact alone, that Theophrastus Bombastus declared war to the knife upon the scholastic degenerate medicine, will assure him our gratitude and an honorable place in the history of the healing art."

Dr. Magnus emphasizes that Paracelsus was himself nevertheless possessed of a medieval point of view, that he attacked his problems and mission not by modern scientific methods but with the same kind of reasoning as was used by nearly all his predecessors and contemporaries, only he discarded the conventional medievalism and sought to substitute a similarly unreal and fantastic natural philosophy of his own based upon neo-Platonism.

"For Theophrastus invented no new weapons but sought

²² *Ibid.*, p. 87.

²³ *Paracelsus, der Ueberarzt*, Breslau, 1906.

to achieve the highest knowledge with just the same equipment which mankind had used up to his time. He thought to discover the secrets of life, of existence and growth, by bold fantastic speculations, just as nearly all natural philosophers and physicians up to his time had hoped to do. So he stands, an embodiment of the conflict which rationalism has waged over the knowledge of nature, at the threshold of the new age—that age which attempts to tear from life its secrets not by speculation, but by observation, investigation, and experiment. Vesalius set himself to the task to bring this new era into the world as Paracelsus, the last romanticist in the struggle over the riddle of life, lowered his blunted weapons and, poor in knowledge, closed forever his tired eyes.”

In these estimates of not inappreciative nor unfriendly authorities, we may understand why it was that during his lifetime, Paracelsus seemed to have so little support among the physicians of his day. It is not perhaps too much to say that the doctrines which he asserted and opposed to the accepted dogmatic medicine owed much of their present interest to certain truths contained in them which were rather intuitively apprehended than clearly conceived by Paracelsus himself. As knowledge grew and facts developed, these foreshadowings which the vision of the Swiss physician perceived rather than demonstrated, gained in authority and respect. It required a later experience to comprehend how much of brilliant suggestion, and prevision of the future methods of science were contained in the thought of Paracelsus.

So while we accord Paracelsus our full sympathy in his unequal battle, we should not misjudge nor too severely condemn the conservative profession of his day, that they did not recognize in him a true prophet of medical progress, but rejected him as a dangerous heretic and mischievous agitator.

Nor, on the other hand, need we be surprised that his native force, eloquence, and the logic and reasonableness of much of his teaching—indeed perhaps even the very imaginative and mystical philosophy by which he sought to formulate his theories of medicine—should have had a gradually increasing influence, so that in spite of the fact that during his lifetime he had few friends and supporters, yet after his death, and as his many writings found their way into print, his work laid the foundation for a very material victory for many of the aims for which he had fought.

Especially it should not be forgotten that, though he seemed to struggle in vain against overwhelming odds during his lifetime—that nevertheless he was largely, if not indeed mainly, instrumental in shattering the confidence of a coming generation in the sacredness and sufficiency of the ancient Greek and Arabian authorities. The remarkable vogue which his writings enjoyed when they were finally printed—the violent conflicts that arose in the profession over the theories and practice he advanced, and which resulted in many victories for the Paracelsan adherents even in the universities, the strongholds of medical conservatism: all evidence that there was great vitality and influence in the ideas of Paracelsus.

J. M. STILLMAN.

LELAND STANFORD JUNIOR UNIVERSITY.

INDEFINABLES AND INDEMONSTRABLES IN MATHEMATICS AND THEOLOGY.

I.

IT is a platitude to say that we can neither define every object nor prove every proposition. But the platitude becomes a somewhat interesting truth if we do not use the phrase merely to remind our hearers of the variable character of the objects known to the world of sense, and the limitations of our experience or sparseness of such records as seem necessary to establish the existence of an occurrence in the past.

Even in the domain of logic we need what Peano has called "primitive ideas" and "primitive propositions". In fact, since all definitions of terms are effected by means of other terms, every system of definitions which is not circular must start from a certain apparatus of undefined terms. It is to some extent optional what ideas we take as undefined. Thus, the idea c may be definable in terms of the ideas a and b , or b may be definable in terms of a and c ; and thus it is allowable to take either a and b or a and c as primitive ideas. Naturally, of two systems of primitive ideas from which the same propositions can be deduced, that system with the least number of primitive ideas would be chosen for logical—though possibly not for pedagogical—purposes. Further, if we wish to make all our assumptions explicit, and to effect the deduction of all our other propositions from these assumptions, it is obvious that the

first assumptions which we need are those that are required to make deduction possible. A proposition can only be proved by means of other propositions. Deduction, then, can only begin by a stating and assuming the truth of the necessary primitive propositions, "explaining" them if necessary. Sometimes their truth is so evident that "explanation" is superfluous, and we at once understand—or at least think we do—though we cannot prove them. We may also "explain" the primitive ideas. The process of "explanation" is psychological: we call up an idea in the mind of the reader by a process which contains the idea to be explained. If this process of explanation of primitive ideas professed to be part of logic, it would of course be objectionable. But it does not: this "explanation" is merely a stimulation of the mind with the object of rousing it to see the idea in question.¹

As regards the self-evidence of the primitive propositions, "the chief reason," say Dr. Whitehead and Mr. Russell,² "in favor of any theory on the principles of mathematics . . . must lie in the fact that the theory in question enables us to deduce ordinary mathematics. In mathematics, the greatest degree of self-evidence is usually not to be found quite at the beginning, but at some later point; hence the early deductions, until they reach this point, give reasons rather for believing the premises because true consequences follow from them, than for believing the consequences because they follow from the premises."

Ernst Schröder³ called the collection of the primitive ideas and primitive propositions of logic the "faith of the logician" (*das Glauben des Logikers*). But, though at first sight there seems to be much in common with the "deposit of faith" mentioned below, we must remember

¹ Cf., for the foregoing, A. N. Whitehead and B. Russell, *Principia Mathematica*, Vol. I, Cambridge, 1910, pp. 94, 95.

² *Ibid.*, pp. v-vi.

³ *Vorlesungen über die Algebra der Logik*, Vol. I, Leipsic, 1890. p. 8.

that the appearance of dogmatism in the logician's procedure is deceptive, since all reasoning without exception assumes some at least of the primitive premises. We may doubt the principles by which we infer, but obviously we cannot make any inference from our doubt. The question as to which principles it is possible to drop for certain classes of deductions has not yet been thoroughly treated. In general, it does not yet seem possible to say more than that some axioms or indemonstrable principles are used very much less than others, so that a refusal to use the former would hardly be noticed by the non-observant.

The principles have been actually formulated, more or less explicitly, and in very different ways, in large parts of arithmetic, geometry, mathematical analysis, and logic, by Leibniz, Frege, Dedekind, Pasch, Peano and his school, Hilbert and his school, Russell, Whitehead, Huntington, Veblen, and others; and many people have considered that the existence of undefinable ideas and indemonstrable propositions in various branches of science is a hopeful sign for theology. Indeed, they think, if theology is reproached by some sceptic for dealing with undefined ideas and unproved or unprovable propositions, it can retort by pointing to what all admit to be the most securely founded sciences of all. "These sciences have now," it would say, "grown up to the stage of frank admission that they rest on a basis of indefinables and indemonstrables; why then urge an alleged insecurity of such a basis against theology?"

Such a thesis seems to have prompted a recent article by Mr. C. W. Cobb on "Certainty in Mathematics and in Theology."⁴ But one somewhat important point appears to be overlooked in such a contention. Whereas the whole of mathematics can be deduced from a small number of primitive ideas and propositions which are necessary for the subsistence of any reasoning at all, it seems that the

⁴ *Hibbert Journal*, Vol. XII, pp. 404-408.

most interesting part of theology consists entirely of indefinable ideas and of—at least deductively—indemonstrable propositions which are none of them necessary for reasoning.

If we admit that there are such things as human beings, that most human beings are not Christians, and that most human beings have some redeeming feature, we can deduce by pure logic—modern logic, not the traditional logic of Aristotle—that at least one non-Christian has some redeeming feature. But such propositions of controversial theology are not of very great interest. The propositions of controversial and other theology which are of interest are such propositions as that the Pope is infallible, that sinless but unbaptized children are saved—or damned, as the case may be—that the Bible was inspired by God, that the Bible's authority is derived from that of the Church, and so on. Now, none of such propositions seem capable of logical deduction from others. In modern logic, indeed, we say that a false proposition "implies" any proposition, true or false, and that a true proposition "implies" any other true proposition. From this it would follow that, if we knew that, in certain well-defined circumstances, "The Pope is infallible" expresses a true proposition, it would be "implied" both by the proposition that the Bible was inspired by God and that it was not, that Socrates died of drinking hemlock and that Socrates died of old age, or by both that twice two are four and that twice two are five. But the logicians do not mean by "logical implication" what ordinary people do, and what ordinary people mean by it seems to be: "Deducible by logical rules from the premises." It is true that we have some reason to believe that these two definitions of "implication" come to the same thing. Thus Dr. X, a renowned philosopher, once doubted that $2 + 2 = 5$ implied that he was the Pope. A distinguished mathematician at once retorted that, since

$2 + 2 = 4$, the premise showed that 5 would be equal to 4, and hence that 2 would be equal to 1. Thus, since X and the Pope are two, they are one. But, unless we *assume* a knowledge of the truth of theological propositions and use the word "implication" in the technical and somewhat paradoxical sense of the logicians, we shall not find, I think, that any important theological proposition implies any other.

II.

Theology, indeed, in so far as it is not ethnology or psychology, would seem to employ logical deduction very little, and to consist of propositions which either describe alleged physical facts or are deductions—not logical, but such as "common sense" draws, in its usual manner, at one time happily and at another unhappily, but often fallaciously, from these facts. Of course, logical rules are used in ordinary conversation. I have heard, with a shock of surprise, some one who had not the remotest idea of what logic is concerned with, apply one of what are known as "De Morgan's rules" with great facility. "Mr. Jones won't believe that Miss Smith is a lady-cricketer!" exclaimed this lively person, "so I suppose that either he won't believe she is a lady, or he won't believe that she is a cricketer!" Still, there is a great difference in the extent to which logic is used in, say, mathematics and that in ordinary conversation.

Logic is the structure in which all our scientific or philosophical knowledge is stored in a readily accessible manner. But the storehouse used for knowledge of the world and of God is very small considering the wealth of information supposed to be stored away. One of the most important functions of logic is to provide a means for arranging our great possessions in facts of nature, for putting these facts in order. We may be possessed of a mass of valuable

papers, but their value is very greatly diminished if they are not arranged according to some easily remembered system; so that we can find any particular paper as easily as we can find, from a knowledge of its number, the house of a friend in an unknown street. If our knowledge of the facts of nature were unsystematized, we might accumulate a very large number of measurements of the corresponding pressures and volumes of a gas at a constant temperature; but Boyle's law makes—or rather would make if it were quite accurate, instead of being merely a good approximation—a unity of all these isolated results. It is evident, by the way, that such a systematization also serves the purpose of lightening the load that memory has to carry, that it “economizes thought,” as Mach puts it; but this important function of systematization is outside our present purpose.

III.

It is a very common custom to attribute strict logical concatenation to the doctrines of the Roman Catholic Church:

“First cut the liquefaction, what comes next
But Fichte's clever cut at God himself?”

“No definition of doctrine uttered by the divine Voice of the Church on earth,” says Mgr. R. H. Benson,⁵ “is in any sense whatever an addition to the original deposit of truth committed to her at the beginning; no *apparent* modification or correction made by the same authority is a withdrawal of any definition previously made; both are alike nothing more or less than a more exact form of statement of the unchangeable Creed of the Church.”

Let us investigate, by way of an example, the premises on which the claim for Papal Infallibility is based. We

⁵ Preface to H. B. Coxon's *Roman Catholicism: An Explanation of Catholic Belief Taken from Official Sources*. No. 32 of “The People's Books,” p. v.

must remember that "the teaching of faith which God has revealed has not been given like a philosophical discovery, to be perfected by human wit; but has been delivered as a divine deposit to the Spouse of Christ, to be faithfully kept and infallibly declared. And it is on account of this that that meaning of the sacred dogmas is perpetually to be retained which our holy Mother the Church has once declared; and that sense is never to be departed from, under the name of a deeper understanding of them."⁶ Again: "It is utterly stupid both psychologically and historically to suggest that when an article is announced explicitly in a creed it is therefore *a new belief*, while, of course, I admit that it is a new *necessary belief*."⁷

All this makes it quite clear that the Roman Catholic Church merely professes to extract and make more definite ("to define") for the needs of the time being the implications of this "deposit of faith." Unless "faith"—besides being "a supernatural virtue, whereby, through the inspiration and help of God's grace, we believe the things revealed by Him to be true, not because the natural light of reason perceives their intrinsic truth, but for the sake of the authority of God Himself who reveals them, who can neither be deceived nor deceive,"⁸—is also a method for drawing necessary conclusions which has hitherto been unrecognized by logic, no intelligent person can admit any justification for the process by which the dogma of Papal Infallibility is distilled from what seems to be a promise which is half a rebuke given by Jesus Christ to Peter.

After repeating that "the Holy Spirit was not promised to the successors of Peter, that by his revelation they might declare new doctrine, but that through his assistance they might preserve holily and expound faithfully the revelation

⁶ Coxon, *op. cit.*, p. 29. This is quoted from an official source.

⁷ *Ibid.*, p. 18.

⁸ *Ibid.*, p. 23. This, again, is quoted from official sources.

or deposit of faith delivered through the Apostles,"⁹ the Vatican Council of 1870 proceeds to reason as follows:¹⁰

"Their apostolic teaching the venerable Fathers have embraced and the holy orthodox Doctors have venerated and followed; fully conscious that this See of Saint Peter remains ever free from all error, according to the divine promise of our Lord and Saviour made to the prince of all His Disciples: I have prayed for thee that thy faith fail not; and thou being once converted, confirm thy brethren. Hence this blessing of truth and unwavering faith was divinely conferred upon Peter and his successors in this Chair, that they might carry out their lofty office for the salvation of all. . . ."

IV.

It is relevant to quote the context¹¹ of the remark to Peter which has just been mentioned. I quote from the Revised Version.

"Simon, Simon, behold, Satan asked to have you [i. e., the Disciples] that he might sift you as wheat: but I made supplication for thee [i. e., Peter] that thy faith fail not: and do thou, when once thou hast turned again, stablish thy brethren. And he said unto him, Lord, with thee I am ready to go both to prison and to death. And he said, I tell thee, Peter, the cock shall not crow this day, until thou shalt thrice deny that thou knowest me."

Accordingly, Peter clearly considered that he was affirming a class of true propositions of the form: "My faith is *now* so firm that. . . .," the "now" being the then present or any future instant; whereas Christ had asserted that at one or more instants in the past, present, or future, Peter's faith was or would be only firm in virtue of the above prayer. Christ thereupon retorted that Peter's faith would fail at a certain future time. All four Evangelists are care-

⁹ *Ibid.*, pp. 73-74.

¹⁰ *Ibid.*, pp. 74-75.

¹¹ Luke, xxii. 31-34.

ful to record both the prediction of Peter's denial and the actual denial, but only Luke records any remark of Christ's about prayers that Peter's faith should not fail.¹² It seems, then, just to conclude that Peter's denial and Christ's foreknowledge of it are at least as certain as Christ's remark that he had prayed that Peter's faith should not fail. Now, a denial involving the shrinking from a possible martyr's death for the sake of trust in the divinity of Christ, was clearly considered by both Peter and Christ as a failure of faith. Thus it is not relevant to say (if anybody should wish to say it—which does not seem very probable) that "failure of faith" does not mean any such human weakness as lying to save one's liberty or life, because of a lack of confidence in somebody else's pretensions; but an inability to perceive the truth of the principles or methods of deduction of Christianity. For, in that case, Christ's prophecy would be quite irrelevant to his "promise" to Peter—a statement of what had happened or was to happen (past and possible future failures of faith but for Christ's prayers) put in the form of a rebuke.

No Roman Catholic, I imagine, would admit that Jesus Christ's prayers were ever ungranted, so this apparently reasonable way of escape is cut off for him. Consequently, he must hold—though he would not, of course, state this belief explicitly—that Christ, when he prayed that Peter's faith should not fail, excluded those future instances (if there were more than one) where it was to fail. That there were to be no such instances after the Ascension, when Peter was presumably left as head of the Church on earth,

¹² It may also be noticed that Matthew (xvi. 18-20) is the only Evangelist who gives the alleged remark of Christ's about Peter, the rock, the Church, and the power given to the Church, which, as everybody knows (cf. also Coxon, *op. cit.*, pp. 66, 72), is a very important part of the foundation of the Roman Catholic Church. We are forced to conclude that a very important part of the Roman Catholic dogma logically depends on the supposition that Christ punned on the name "Cephas," and that Matthew or the unknown author of "Q" translated Christ's pun. (Cf. Hastings' *Dictionary of the Bible*, p. 759.)

is nowhere stated, and is not logically deducible from the proposition that at some instants Peter's faith was not to fail. The extension of the freedom from error to Peter's successors again requires some other than a merely logical principle for its deduction from the "deposit of faith."

So long as Peter's faith did not fail—which we will assume, for the sake of argument, was always the case until the denial—we may hold that Christ's prayer was certainly granted, and we may even hold, it seems, that Peter's faith would have failed if Christ had not prayed. But, the moment that we extend the space of time forward, so as to include at least one moment when Peter's faith did, in fact, fail, we must conclude that Christ's prayer could be reduced to the prayer that Peter's faith should not fail except when it should happen to do so. Now, we cannot suppose that Christ supplicated for what would happen, by logical necessity, whether God existed or not; so we are driven to the assumption, if we wish to hold that Peter's faith remained firm after, say, the Ascension, that Christ's prayer excepted, by defining them, the occasions of failure. Hence, to prove that Peter's faith did not fail (according to Christ's prayer) after a certain time, Roman Catholics are obliged to assume that Christ's prayer was that Peter's faith did not fail after that time. Now, if p is a proposition, p certainly implies p ; but it is rather surprising to find a Papal decree being made of the fact that, from the "deposit of faith" (p, q, r, \dots), is deduced, not a proposition which is implied, but not always recognized to be implied, by (p, q, r, \dots), but one of the old propositions, p itself. If Papal decrees are disguised identities, one can readily understand why they are infallible.

V.

It is conceivable that a Roman Catholic might urge that belief in a divine promise about Peter's faith is not derived

from
obj
con
exp
fro
bel
be
"de

use
for
pra
of
no
insta
faith

from the Bible alone, but from tradition. But such an objection is irrelevant to the present argument, which is concerned with the deductions which the Vatican Council *explicitly stated*, in the passage I have quoted, to follow from Luke's report. Surely the statement that the Fathers believed in something "according to" Christ's words cannot be ambiguous. If so, it is clearly a Papal duty further to "define" Catholic doctrine.

VI.

The points of the above argument are as follows:

1. Peter's denial was a "failure of faith";
2. We assume that Christ had prescience and that his prayers were always granted;
3. Propositions (1) and (2) imply that Christ must have prayed that Peter's faith should not fail except on certain occasions (one or many), which he must have specified;¹³
4. Consequently, the deduction that Peter's faith did not fail after a certain time requires itself as premise;
5. Consequently, the doctrine of Papal Infallibility is either part of an identical proposition, or else Christ must be assumed to have prayed that an identity may be fulfilled.

It seems, then, that, if we are to be able to attach any useful meaning to Luke's narrative, we must adopt some form of the doctrine of kenosis, and admit that Christ's prayers were no more likely to be fulfilled than the prayers of ordinary mortals, and that Christ's prescience was by no means complete.

¹³ Unless Christ prayed that Peter's faith should only fail once. If the instances are quite unspecified, Christ's prayer must have been that Peter's faith should not fail except when it should happen to do so.

VII.

In the Rev. F. H. Chase's article on "Peter (Simon)" in Hastings' *Dictionary of the Bible*, the following account of Christ's promise is given (p. 761): "But Peter had been the subject of urgent supplication on his Master's part that his faith might not wholly and finally fail (ἐκλίπη). It is implied that the Apostle would not pass through the trial unscathed. But beyond the trial a return to former spiritual relationships is promised—a return which would bring with it the duty of 'stablishing his brethren.'" On the same page, Peter's denial is spoken of as "faithlessness"—a fact that confirms our first point.

This interpretation of Christ's prayer is certainly more conformable to a later event, where Peter, according to Paul,¹⁴ was, in Principal Chase's¹⁵ words, "guilty, not of false doctrine, but (as once before) of moral cowardice," and consequently, according to the same writer, of "faithlessness." But it is quite obvious that this more reasonable interpretation of Christ's prayer is quite fatal to the claim for Peter's absolute firmness in faith and infallibility after the Ascension.

VIII.

In what precedes, it has not been my intention to depreciate religion simply because of its very sparing use of logical deduction and overwhelming number of indefinables and indemonstrables. There seems, indeed, some ground for objection to the admission of primitive ideas and propositions which seem to be partly arbitrary and partly based on very slender evidence—the report, for example, derived from unknown sources, that somebody else alleged such and such a thing to have happened, other evidence for the

¹⁴ Gal. ii. 11. It may be noted that this Epistle is accepted as canonical by the Roman Catholic Church.

¹⁵ *Loc. cit.*, p. 765.

happening of the thing being somewhat conspicuously absent. But everybody must admit the nobility of the straining after ideals which neither help us to attain bodily comforts nor put us in the way of gratifying any of the very common and mundane desires of human beings. Such an ideal was the intellectual "curiosity" of which Aristotle¹⁶ speaks; such ideals are those of many, both now and in the past, who have made sacrifices of themselves for what they believed to be the truth; such are the exalted ethical ideals of religious persons. It is a matter for wonder and thankfulness that such ideals exist, even if a primitive credulity and hankering after magic seems always to accompany them at first. This is by no means the case with religions alone. The sciences still bear numerous traces of that primitive credulity which would make a "law of thought" of the principle of the uniformity of nature, and — as Mach and Karl Pearson have shown — of the fetishism involved in such conceptions as that of "force." Certain people have tried to abolish or explain away—in part at least—the "miraculous" element in religions, and bring religion more into harmony with philosophy and science. But a deductive theology is still a very distant ideal.

PHILIP E. B. JOURDAIN.

CAMBRIDGE, ENGLAND.

¹⁶ See Prof. A. E. Taylor's *Aristotle* (No. 67 of "The People's Books"), p. 14.

A CATALONIAN PHILOSOPHER: ANTONIO
COMELLAS Y CLUET.

THE name of Antonio Comellas y Cluet will always have a distinguished place in the history of philosophy, especially that of Spain, where he was deemed a laborious and illustrious thinker. He is one of the Spanish thinkers whom I was not able to overtake or discuss in the chapter devoted to "The Philosophy of Spain" in my *Studies in European Philosophy* (Blackwood, 1909), and this paper is devoted entirely to him in supplement of the treatment then made of churchly-scholastic philosophers in Spain during the last century.

Comellas found himself planted in a time in which the negations of positivism had found their way, and the critical effects of Kant had been felt. It was this atmosphere that made him feel the need to consider the possibility of science, and the legitimacy and value of scientific method and procedure. This explains the character of his great work, *Introduction to Philosophy*, which is no mere propædæutic, as the phrase might suggest, but a critical study of method; of the sources of knowledge; of the criteria of truth; of the motives or reasons of certainty; and of the bases of science, under rigid analysis. I wish to say that, in addition to my own study of the work of Comellas, I have had the advantage of the valuable study of Comellas made by another distinguished Spanish philosopher, Alberto Gómez Izquierdo, in *Cultura Española* (Madrid,

1907). Comellas cannot be said to have had the originality to create an entire philosophical system, though a synthetic organization may be claimed for his thought and its discussion of fundamental positions. Some account of these may be of interest, and I shall deal only with those points that seem most essential, making such critical comments as seem to me necessary. He did not found, and scarcely affiliated himself to, a particular school.

I.

It may be well to consider first the position of Comellas on the methodological problem. He makes three stages or periods in the evolution of science. The first of these is that of the *real*, in which man holds to being or to forces, without as yet having unfolded them. He emerges from this condition through the attractive power of the ideal, and the aspiration for truth. Comellas has, rightly I think, emphasized the emergence of the opposition of fact and ideal—as various thinkers have done since—but we must not lose sense of the slowness of the process in the crispness of the statement. Nor must we overlook, in speaking of the ideal, the retardations due to the slow-footed movements of the positivist spirit in various spheres of scientific knowledge. The second stage is that of the *attracting ideal*, wherein is seen the activity of human reason in abstraction, generalization, induction, and deduction. In the third stage, we have *the synthesis of the real with the ideal*. On this hypothesis has Comellas based the formation of science, grouping around it the various questions as to critical method. Totalizations for mind is thus what Comellas derives from the fusion of empiric fact with intellectual working and ideal. It should be observed that the three stages now enumerated form the three divisions of the work.

The position of Comellas as to universal doubt being

the point of departure in scientific investigation is, that such doubt is a skeptical means to a dogmatic end (p. 33). He does not think even partial doubt is necessary as an antecedent of investigation. Nobody, he thinks, would contradict the fact that as often as we investigate things, we doubt concerning them. And yet we can, in his view, frequently see that people investigate things without doubt of them, and he gives examples of such cases (pp. 67-69). One of these is the case of trying to demonstrate an accepted truth by new arguments. One thing, I may remark, that cannot be doubted is that we doubt. There is a rational skepticism which is necessary and healthful: there is also an enervating skepticism, which weakens the mind's trust in itself. Whatever scientific knowledge or certainty may have owed to doubt, it should not be so often overlooked how much more it has owed to rational belief—belief, I mean, in those rational, self-evident, necessary principles which make fundamental appeal to our rational insight. It has been said by another distinguished Spanish philosopher, Balmez, that "certainty does not originate in reflection; that it is the spontaneous product of man's nature, in the direct working of his intellectual and sensitive faculties; and that philosophy simply examines the grounds of certainty, while recognizing that those first principles, which are the basis of all evidence, are themselves incapable of proof" (*Fundamental Philosophy*, Bk. I, Ch. III). But I may add that direct and natural certainty is, to scholastic philosophers, the basis of philosophic certainty, which is of reflective and demonstrative character. For Comellas the certain and the evident are but the support necessary to soaring toward the ideal. But the ideal is not to be sought in the works which our predecessors have left us. To no writer or generation has Deity said, Thine are the treasures of science, gather them according to the measure of thy mind, and they will be the unique patrimony of

humanity (p. 81). In this Comellas spoke wisely enough, for no attempts to coordinate all human science, whether those of an Aristotle, an Aquinas, a Descartes, a Leibniz, or a Comte, can have finality, though every one of them must have great value and most helpful function. Comellas quotes interesting passages from Anselm, Albertus Magnus, Aquinas, and Roger Bacon, setting forth the doctrine of progress in science, which is thus not so entirely modern a doctrine as is often supposed (pp. 82-85). Not through our predecessors is the ideal found, but as the result, Comellas thinks, of our own search, with amplitude of thought, personal examination of issues, and scrupulous rigor of observation, taking due account of the workings of reason, and the evolution and conditions of our faculties of knowledge, whether empirical, or abstractive, or deductive (pp. 108-109). These three moments Comellas treats in this successive order: perception of an object, abstraction, and contemplation of the object abstracted; intuition of its content; and, finally, combination of the content of the object abstracted with empirical fact, wherein consists deduction. All these acts are discussed by Comellas at length, but we have only room for a few observations.

Perception he discusses in its differences as related to the corporeal world, and the phenomena of spirit; the former he refers to sense, the latter to consciousness. In respect of empiricism, he finds the methods of Mill useful. This does not keep Comellas from being independent and critical of Mill, as he is, for example, on induction (pp. 184-194), where he objects to Mill's positivist tendency in eliminating the metaphysical element from deduction, and making all the deductive sciences inductive. I may observe that there is no need to distinguish induction and deduction so sharply as is sometimes done, as they are so closely inter-related; and that critics of Mill like Jevons and Welton recognize the necessity of deduction in respect of the con-

sequences of the inductive hypothesis as first framed. We can no more dispense with the deductions of Laplace, Lagrange, and Gauss, than with the great induction of Newton. The fruitful results of induction for knowledge must not obscure for us the splendid scope for intellectual power afforded by deduction, with its expansive effects for knowledge. Infinite being or substance purely spiritual, cannot, Comellas thinks, be direct object of abstraction; it is reached by combination of abstractions. He takes a wide view of intuition, including in it various dialectical processes; and to intuitive power and working, not to inductions based on fact, he attributes the formation of universal principles which are absolute and evident, and forms the most important factors of deductive thought. Ueberweg's *Logic* has had influence on his view of deduction, and on other matters (as for example, later, at pages 233 and 243). The foregoing matters are discussed at considerable length (pp. 155-194), but cannot here be further pursued.

II.

Then Comellas dealt also with the critical problem. He took large account of psychological processes in our powers of knowing, of the motives of human certainty, and of the nexus between idea and reality. In objective evidence he found the supreme reason of certainty, and the sure criterion of truth. The objective reality of the object as perceived was, for him, indisputable. You could not find either activity or receptivity in nothing. To say that a thing is perceivable, and does not exist, is to affirm a contradiction (p. 111). The relation between perception and reality he held to be of the most intimate kind, so wrapped in the apprehension of the object indeed that we become certified of its existence. He deduces the reality of the object, it should be said, from the application of the principle of causality to the affective sensations, but the knowledge of

the object is not direct or immediate, as in the case of perceptions. In the position outlined in the earlier part of this sentence, Comellas seems to follow the medieval usage of speaking of the causation of sensation by the object; the confused and perverted medieval meanings of the phrase *species sensibilis* in that connection did so much to obscure the apprehension of sensation as a mental fact, that Comellas had done better, I think, to avoid causal emphasis, and to extricate the mental fact of sensation from physical, and set it in psychological associations. In the latter part of the sentence, the position of Comellas is much in accord with that of Maher, who says that "a sensation is in itself an elementary mode of consciousness of a cognitional character" (*Psychology*, p. 46). On the question of sensation, Comellas was largely influenced by the scholastic philosophy. Thus he held that we perceive sensible objects only by touch and sight, the other senses being affective, and by them we experience only a subjective affection, caused by the objects (p. III).

Thought is one thing, says Comellas, the proposition "I think" is another (p. 237). He treats the truth and reality of knowledge of the outer world, by means of apprehension, as absolute and indefectible, but the truth and exactitude in the other case, where the data of consciousness are concerned, he ranges under the formula "I think" as a mode of locution more or less accurate—in fact, secondary and derivative. There seems to be some lack of clear discrimination about his treatment here. There may, no doubt, be error or inadequate apprehension when principles come to be applied, and opinion comes into play, but the function of reason in judgments should not be overlooked, nor should the fact be forgotten that there are both self-evident moral axioms, and self-evident and necessary truths of reason or intellect. And self-evidence, it has been said, "is really the only possible ultimate test of truth, and

must be accepted under pain of complete intellectual paralysis. It is incapable of demonstration, since it depends on nothing else" (St. George Mivart, *Elements of Science*, p. 381). Neither in the case of self-evident truths of intellect, nor of self-evident and necessary moral truths or principles, can it be satisfactory to talk of secondary and derivative, in the language of Comellas, since they assume a character necessary, universal, absolute. This, of course, does not supersede, in the case of necessary moral truths, the need for moral training and development, in order to eliminate error and perversion in the way we hold them, and to perfect fulfilment of the functions of conscience. Comellas seems to me to connect all certainty too much with sense-perception. But there is no pure sense in man without some act of the understanding; it is never purely *ἄλογον* (Aristotle, *De Anima*, III, 9, 2). And I hold that there are no good philosophical grounds for the idea that sense-knowledge is more valid and direct than other knowledge. Comellas concludes, somewhat summarily and realistically, that the formula "I think" cannot be the point of departure for philosophy, since it does not relate to what he calls primordial knowledge. Now, of course, the formula "I think" cannot, as he rightly says, be the point of departure for philosophy, since it is but the ego reduced to the pure form of unity, but because this is true of Kant's blank form of thought or pure *cogito*, there was no reason why Comellas should not have better appreciated the aspects of moral certainty, the justice rendered by Kant to the moral reason, and his claims for moral value. The natural realism of Comellas seems to me to have crowded out, to too great an extent, his moralism. And so it happens that to-day we even see the emphasis of Comellas inverted by some thinkers, who make the conception of intrinsic value as found in Kant's ethics bear the weight of the whole philosophic structure. But creative idealism

cannot, as it seems to me, be confined in its working to such a narrow moralistic basis, but must include in its purview the whole world of reality, fact, imagination, and thought. Meanwhile, the conception of intrinsic moral value as the clue to the ultimate nature of reality has not been shown capable of being carried out into anything like a valid or satisfying world-view. In such a world-view, both this moralistic procedure, and the nature-knowledge of Comellas, must find place, but neither of them in overweighted or one-sided manner.

Comellas takes the multiplicity and succession of spiritual acts to be facts or undoubted reality for consciousness, in which result he thinks the Kantian hypothesis of the subjective forms *a priori* of space and time to be inadmissible. He admits a correspondence between the object perceived and our idea of it, but thinks it is not absolute, and does not exhaust the reality of the object, but remains part of its content (p. 154). I must remark, however, that neither does the subject exhaust itself in this reference to an object, as a fact to be equally recognized in psychology. In the case of infinite being or pure spirit, such correspondence of the concept with the real is not necessary, he thinks, since it is no case of an object perceived, but a procedure by combination of abstractions. It may correspond to a real being by deduction (he thinks it does so in the case of infinite being), or it may be merely possible being. The objective truth and reality of our knowledge Comellas firmly holds; he thinks our means of knowing are a legitimate source of truth; neither sensible representations nor consciousness-data nor abstract conceptions are, to him, the exclusive creation of our thinking activity; they are resultants of the knowing subject and of the objects which determines and informs it. He thinks that in man the possibility of error springs from his limitation and his liberty (p. 212). The will can determine the judg-

ment, but may do so wrongly, as it lacks security—a good hint to voluntarists. Anyhow, limitation and liberty are the immediate source of error in man (p. 213). Perhaps Comellas was here influenced by Descartes, to whom he devotes chapters in the second and third parts of his work, for Descartes made error consist in defect or finitude—our “faculty for discerning truth from error” is “not infinite.” In the discussion of error, Comellas thinks certainty involves the persuasion of the truth, but not the truth itself; certainty must justify itself at the bar of reason, to which latter it must conform. The rational motive of certainty and the criterion of truth as distinguished from error, must, he opines, be found in objective evidence, not in clear ideas as Descartes supposed, unless it be in clarity of the object as perceived or seen. When discussing the view of Balmez, of which he is somewhat critical (p. 223), he sums up his doctrine on the criterion of truth in the formula: “The evident is true” (“*Lo evidente es verdadero*,” p. 222). This principle of evidence, Comellas insists, “comprehends all things evident, be they classes, or propositions, or facts, or substances, or accidents known by act of vision” (p. 222). What Comellas is, I think, really trying to bring out, albeit not so cleverly as might be done, is, that certainty is the concern of all knowledge, and that in all certainty, whether of what is external or internal, there is a real object; but he does not bring out, with sufficient clearness, that, while the general form of certainty is the same in all knowledge, the variant forms of certainty—mathematical, logical, scientific, moral, spiritual—are due to the differences in the matter involved. Comellas disposes somewhat lightly of the objection to his position that the object perceived has reality of being, by saying that, in the case of illusions and hallucinations, there is only sensitive appearance, not a perception properly so called. In all such cases of mere appearance, there is “no

evidence or objective perception" (p. 229). What he defends is "the truth of the evident, the perceived or seen, but not of things apparent." False or imaginary appearances presuppose "the truth of the evident, of the perceived or seen" (p. 230). Earlier, he has insisted on the principle "the perceived is true" (p. 225) as like unto what he had said of "the evident," though he admits that, strictly taken, evidence and perception must be distinguished. Comellas, in all this somewhat hard realism, does little, it must be said, in the interest of the flow of free ideas, and the unifying of the perceptual data. It is evident that Comellas has not a little in common with what Professor Sidgwick called "the earnest, patient, lucid, and discerning intellect" of the Scottish philosopher Reid (*Mind*, 1895), to whom Comellas devotes a chapter in the third part of his work. There are some interesting differences, no doubt, but on these we need not now go out of our way to dwell. It must suffice to say that, though Comellas thinks we have means enough to distinguish the truth from error, he does not think we can fully reach the scientific ideal.

III.

Lastly, there are the critiques of Comellas on philosophical systems. These I have thought it desirable to place in a third section by themselves. But it should be observed that they belong mainly to the third "book" or division of the work of Comellas, which deals with the synthesis of the real with the ideal in twenty-five chapters. The critiques are introduced by half a dozen chapters on subjects like truth, error, evidence, certainty, and appearance, and I have spoken of those points in them which I wished to notice under the critical problem, as the most appropriate place. But it is in reference to the principles and criteria in them that the philosophical systems are examined and discussed by Comellas. The critiques are, it must be said,

fragmentary, not exhaustive, but the critical work in them was done with scrupulous care and fairness, giving in footnotes the *ipsissima verba* of the thinkers concerned, whether German, French, or English, whenever necessary. Among the subjects of his critiques are, the methodical doubt of Descartes; the teachings of Reid, of Balmez, of Jacobi, of Lamennais; the skepticism of Kant, the relativism of Hamilton, Comte, and Spencer. Aristotle, Carneades, and Sextus Empiricus, have some attention, too. Some discussion of Plato and Aristotle, and also some treatment of Hartmann's philosophy of the unconscious, had already been given in the first "book." The pessimism of Schopenhauer; the eclecticism of Cousin; the metaphysical ideas of Krause; the views of Mill on induction and the nature of axioms, had all received handling in "book" second. Comellas, in his treatment of the systems, seeks to find in them confirmation of his own criterion of the true. I shall only speak of some points which seem to me of present-day interest.

Sextus Empiricus is repeatedly referred to by Comellas in connection with skepticism, a subject to which he devotes much attention. Comellas quotes (p. 305) from the *Pyrrohonic Institutes* of Sextus, wherein he says that those who think the skeptics deny phenomena do not understand their doctrines. Sextus says they do not reject things which by the impression they make upon our minds induce in us an irresistible assent. Such are phenomena. But when we examine whether the object is as it appears, we concede that it appears, but doubt whether it exists in the manner that is affirmed. Thus we concede, says Sextus, that honey has a sweet taste, because we feel sweetness, but nevertheless doubt that honey is sweet, according to the dictate of reason. With such teachings of Sextus, Comellas connects the teachings of Kant concerning the phenomenal and the noumenal, pursuing in a critical man-

ner Kant's denial of the possibility of knowing things-in-themselves (pp. 305-315). He further quotes Sextus, who says that the skeptic does not employ the expressions of doubt in an absolute sense; he neither says that "all things are false," nor that "nothing is true," without recognizing that each of these statements has its own falsity. Sextus speaks similarly in his work directed against the dogmatists, and entitled *Adversus Mathematicos*. The affirmation of universal doubt thus goes united to doubt of that affirmation itself (p. 309). In this connection I would observe that doubt itself rests on certitude, and that, to put the whole matter more explicitly, absolute skepticism is really self-contradictory.

Comellas points out that, in the seventh book of this latter work, Sextus holds that no firm seat can be found for the negative doctrine that there exists no criterion of the truth, since to the reasons advanced for that position others equally probable can be opposed. Firmly to suspend all assent is the conclusion he reaches (pp. 308-309). Comellas also cites the passage from the eighth book of the same work in which Sextus urges against the dogmatists that, to hold that to know the truth of a thing involves necessary, infinite, and impossible suppositions, would mean that one could not with certainty know the truth of anything (p. 309). Thus there and elsewhere Comellas thinks there are indications in Sextus which point to the need of some certainty. Indeed, later, Comellas finds that Sextus at times inclines to certainty in all three moments, the empirical, the abstractive, and the deductive. This would detach him from the position of the skeptic, viewed as one "holding no form of creed but contemplating all." Ritter says that, in all this, Sextus does not give "his exposition of the skeptical doctrine as anything new, but invariably speaks in the name of his school" (*History of Ancient Philosophy*, Vol. IV, p. 273). It has seemed to me more

important to dwell on the points connected with Sextus, both because they are well chosen by Comellas, and a good edition of Sextus Empiricus still remains a desideratum, and because "the skepticism which prevailed in Europe from the beginning of the sixteenth to about the close of the eighteenth century drew its inspiration, its principles, and its methods, largely from the writings of Sextus" (R. Flint, *Agnosticism*, p. 95).

As to Descartes, Comellas is critical of what he considers his subjective and skeptical tendency. But in his famous axiom, *Cogito ergo sum*, Comellas says Descartes expressly excludes from doubt his own existence because contained in his thought, and "thought," for Descartes, comprehends, he says, all the acts perceived by consciousness, save those which are by means of the bodily organism. Under "thought," moreover, are included, not only understanding, willing, and imagining, but also *feeling*. No doubt, the principle had a subjective and personal significance for Descartes, but it was, I think, rather meant, through the involved perpetual presence of thought, as a refuge or defense against skepticism, when, in the *Method*, for example, he says, "The 'I,' the mind by which I am, is wholly distinct from the body, and is even more easily known than the latter; although the latter were not, it would still continue all that it is." Descartes points out that his *Cogito ergo sum* is not a syllogism, but rests on a simple intuition of the mind (*simplici mentis intuitu*). And one need hardly say that any real doubt of our own existence would mean mental paralysis. Comellas objects that Descartes, in his discussion, confounds conception and vision: he says Descartes speaks of vision, which is a fundamental act, and then passes on to speak of conception, which is derived from it. Comellas at some length criticizes Descartes's psychology on other matters; he thinks Descartes fails to distinguish apprehension of the object

from the mental expression of it. For my own part, I doubt whether the famous axiom of Descartes is so absolute as is commonly supposed; and the possibility of its conclusion depends on the truth and certainty of our intellectual operations. In making the "I think" involve and express the "I am," the axiom seems to become, in some real sort, a *petitio principii*. It seems to me noteworthy that Augustine and Aquinas had already used it in principle—the latter expressing it very clearly in his own way, and in a relative sense—but they held to self-consciousness as primitive indubitable fact. Yet Descartes is generally spoken of as if he were the first that ever entered the truth here, when what he did seems rather to have been to give it new, distinct formulation. It may be allowed, I think, that the axiom brings out that conscious personality is involved in every mental act.

In treating of the Spanish philosopher Balmez, whom I have dealt with elsewhere (as indicated at the beginning of this article), Comellas properly notes the influence on him of Descartes in respect of subjective tendency, and the influence of the Scottish school, as to the grounding of certainty in assent through irresistible inclination.

Comellas regards Reid and Jacobi as occupying similar positions on the principle of certainty. This Reid found in instinct or irresistible inclination, so emphasizing the "primary instincts of nature," and undervaluing the reflective reason; Jacobi found it in feeling, for to him reality can only be believed in and felt, not known. In both cases, the principle consisted in something distinct from knowledge. Jacobi's basis in feeling, Comellas thinks, is a weak one for the grounding of science (p. 286). He thinks like faults attach to their systems; they disavow, in part, the harmony of the universe; and they do not provide adequate safeguards against skepticism. While these general conclusions of Comellas are interesting, I add a few words for

the sake of clearness. Reid and Jacobi were alike in holding an immediate knowledge before all mediate knowledge, before all inference, and all fixed and fundamental truths. Reid found a place for self-evident and intuitive truths. "There is no searching for evidence; no weighing of arguments; the proposition is not deduced or inferred from another; it has the light of truth in itself, and has no occasion to borrow it from another." He distinguished necessary from contingent truths. Though his philosophical analysis was defective, his philosophy was yet a safeguard against subjective phenomenalism. Jacobi developed his immediate knowing, which is feeling, into the two forms of perception and reason, the former for the sensuous, the latter for the supersensuous. Not through thought, but through feeling, does the actual assert itself, for Jacobi, in human consciousness. Jacobi is not like Reid, when, for him, there is no reality outside consciousness, but he reminds us of Reid in his insistences on immediate perception. They both fought for the rights of immediacy and of reality, but each in his own way. Comellas thinks skepticism has been shown not to have foundation enough to engender certainty of the truth (p. 323). In which connection I would recall what was well said by Dr. Edward Caird, that "the very logic by which the skeptic overthrows the dogmas of philosophy, implies that the mind possesses in itself the form and idea of truth. His deepest doubt reveals a certitude that transcends and embraces it."

Lamennais is criticized by Comellas for his advocacy of the impotence of human reason to distinguish the true, and to reach certainty, and for his impressive attempt to found a doctrine of traditionalism on skeptical bases, or, in other words, to take refuge in divine revelation. This, Comellas argues, would make the criterion of the true objective-subjective, and set it in an extrinsic order. Not only does Lamennais make divine revelation the principle

of certainty, but, sacrificing the individual reason, he makes the common or catholic consent the means of knowing that revelation. The infallible guarantee of the primitive traditions is found by Lamennais in the human race. Divine authority is made by him the basis of the certainty of our knowledge. Common consent or collective reason is the only seal or stamp of the truth. But, in the view of Comellas, the doctrine of Lamennais has no solid foundation, since it carries no immediate evidence, nor any demonstrative force. It is impotent, he says, to reach certain knowledge of the truth, since it does not proceed from individual reason. The common consent was, to him, a begging of the question, a supposing of the very thing to be proved. The doctrine of Lamennais was, to Comellas, self-contradictory. It assumed individual reason able to be certain of the common consent as the means of knowing the divine revelation; it did not allow individual reason to reach certainty of any particular fact, but supposed it able to be certain of so general a fact as the common consent. I will only remark that Lamennais carried his skeptical basis to an extreme that was self-destructive. If you begin by making individual reason wholly fallacious, and treat individual assent as of no value, the general consent, built on such a basis, can have no value either.

Comellas has a good chapter on the philosophy of Krause, who has attracted much attention in Spain. But Comellas objects to it as giving an excessive importance to experience, and as participating in the subjection of Fichte and the intellectual intuitionism of Schelling. His criticism is based on study, not only of Krause's own works, but also of Tiberghien's exposition of Krause (in his *Introduction à la philosophie*, Brussels, 1880). The final conclusions of Comellas are, that Krause's philosophy, on one side, exaggerates the powers of the human understanding by admitting a single principle of all our knowl-

edge; and, on another side, is opposed to its full development, inasmuch as it adopts an insufficient point of departure, making the contemplation of being the fundamental principle of all knowledge, and neglecting the empirical element in its deductions. For these reasons Comellas cannot allow the philosophy of Krause the merit of the extension, or the purity, or the solidity, of science or knowledge. Still, I do not think we can forget that it had its distinct merits, not the least conspicuous of which was his fine insistence on objective morality in law and history. But it was a mistake when Krause made nature and reason or mind coordinate, instead of, like Hegel, making nature subserve the ends of reason or mind.

Comellas is critical of Cousin's eclecticism, with its claim, by this method, to reach the whole of truth. In this I need hardly now follow him, except to say that I think he took a too external view of the elective method—as so many have done—and this has been very clearly corrected by some recent French philosophers. I think it more needful to remark that, in the view of Cousin, truth was always present in reflection, although there might enter error too. And another point, not overtaken by the limited treatment of Comellas, is the interesting distinction raised by Cousin between spontaneous and reflective intelligence, but not satisfactorily disposed of by him (see R. Flint, *The Philosophy of History*, pp. 461-464).

Comellas objects to the relativity theories of Hamilton, Comte and Spencer, which he thinks experience does not favor (p. 370). Certainly it was absurd when Hamilton and Spencer thought we cannot know the object as it is in itself, on the ground that we cannot know it save as different from and yet related to the subject. Absurd, because knowledge that should not involve distinction is really impossible. The want of complete knowledge, which holds true of all reality, does not involve ignorance of all the real

(p. 370). It is too often overlooked, I must observe, that, if the doctrine of the relativity of knowledge be true, it must be true of the doctrine of the relativity of knowledge itself. If it is, as a theory, merely relative and phenomenal, then it cannot be known to be true. But if it be held as really true—which means that some of our knowledge is absolute—then it can no longer be true to say that all our knowledge is merely relative. Knowledge is, indeed, in a proper sense, relative, but it establishes a relation between us and the absolute, whose nature is self-revealing. While skepticism, in an absolute sense, is self-destructive, its method is not without real advantages in scientific investigation, where theoretic assumptions must be tested. I think Comellas overlooks too much this aspect of the case. The impossibility of understanding the most recondite forms of being, to which Spencer refers, proves the limitation of human knowledge, but does not, in the view of Comellas, justify skepticism (p. 370). With sure knowledge of substances, acts, and real attributes, may well be reconciled, in his view, the want of unlimited knowledge. The human understanding has, in his view, the force of which these philosophers deem it destitute (p. 374). In the three moments, empirical, abstractive, and deductive, he holds (pp. 360, 374) that we know things-in-themselves, and not mere appearances. I am tempted to add, in this connection, the words of a forgotten writer on ethics, "Men have always chosen skepticism before absurdity, when both are fairly before them; because skepticism, though itself the greatest absurdity, is of all others the most recondite and imposing." Comellas thinks the real world does not bear the sombre character which skepticism attributes to it. The harmony and mutual expansion obtaining, in the empiric moment of our knowledge, between man and the world's forces, and the vigor evidenced by man in the moments abstractive and deductive, are a manifestation of the ten-

dency of the universe toward the ideal (p. 375). The universe, says Comellas, is not a dark abyss, into which the gaze of man cannot penetrate; it is, on the contrary, inundated with light, and allows itself to be scrutinized by our limited intelligence.

In conclusion, I may remark that such discussions as those of Comellas, with their stress on the objective aspects of truth and certainty, raise issues too much overlooked in modern subjective theories. To take one of the latest of these—that of Croce—truth is never static, to Croce, not even in logic, truth is faith, certainty of one's self, free development of one's inner power. But this subjective trueness of one's spirit can be no substitute for, or replacement of, the objective aspects of truth and certainty. If truth is faith, Croce's pure concepts are pure ideals, for faith is always an unfulfilled ideal, and truth is then never realized. But the science of Being, or the metaphysics, which Croce has discarded, is not to be disposed of in any such way, but remains the permanent core of all immanent evolution.

IRVINE, SCOTLAND.

JAMES LINDSAY.

LOT

T
tire o
been
than
recen
terest
this s
egori

It
that v
comp
and a
bearin
to inv
begin
is in t
a.
"empt
.... a
of its
crete"
with

¹ M

LOTZE'S THEORY OF THE SUBJECTIVITY OF TIME AND SPACE.

THE philosophy of the ultimate nature of time and space determines, to a greater or less degree, the entire conception of world-order; and this again has rarely been given a more systematically thoughtful consideration than by Lotze in his *Metaphysics*. Perhaps the trend of recent thought toward some type of realism may lend interest to a brief examination of the grounds adduced by this severely logical thinker for regarding these two categories as subjective.

I.

It may serve to orientate the discussion to remark here that while Lotze agrees with Kant in taking space to be completely subjective, he dissents from Kant as to time; and as this difference appears to me to have an important bearing upon his final standpoint it will, I think, be best to invert the order of Lotze's own treatment, and so to begin with time;—here exposition, rather than criticism, is in the main sufficient.

a. To begin with, then, Lotze discriminates between "empty time," taken as having "an existence of its own . . . as a power prior to all reality and governed by laws of its own,"¹ and what may perhaps best be called "concrete" or "filled time"; and here I think few will disagree with his view that the former entity is subjective; sub-

¹ *Metaphysics*, I, p. 320.

jective, however, in the sense that such empty time is an abstract conception—"a creation of our presentative intellect. . . . when we think of the lapse of time. . . . and make abstraction of the content (*loc. cit.*, I, p. 350). Of this—the empty total image of that order in which we place events as a series—it is true that it is only a subjective form of apprehension" (p. 354). In this formal sense, "there is no real time in which occurrences run their course" (p. 343); but nevertheless (and it is here that Lotze differs from Kant) time, from another point of view, is essentially real; for "of the succession belonging to (the operation of things) itself, which makes the arrangement of events possible, the reverse is true—it is the most proper nature of the real" (p. 354). And while the problem of time remains one of the most difficult in philosophy, still we need claim no deeper reality for formal and empty time than for any of the abstract conceptions of, e. g., pure mathematics.

But at the same time, though abstract, and in that sense subjective, it is not therefore wholly fictitious; it retains, as Lotze himself points out with regard to space (p. 258), an indubitable reality of its own. It is, thus regarded, a device, or schema, by means of which its originator, the finite mind, enables itself to comprehend a reality too complex to be seized in all its fulness; and if we take mind to be throughout all its manifestations one in its nature, then we must hold that it is always possible to endow the abstract time schema with deeper reality in one of two ways—either by filling it with concrete process-content, or by somehow attaining to a more absolute vision of reality as a whole. And therefore to say with Lotze, regarding this schema, that "there develops itself the idea of time" (p. 343), by no means implies that this idea is only, as Lotze himself regards it, a "fantastic image" (p. 350), but rather asserts the evolution of an organ of mind, which is fully analogous to that of any bodily organ; for mind is

orga
and
are a
point
as a
to be
b
come
acco
their
cond
the c
quen
F
the f
cogn
prin
past,
from
consc
diti
cond
possi
follow
cogn
simp
mode
inter
tions
seque
of a
that
that
condi

organic like the body, and rarely generates abnormalities; and as in biology, so in psychology problems of function are associated with those of development; but just at this point, perhaps as a consequence of regarding formal time as a "fantastic image," Lotze's further treatment appears to be not wholly adequate.

b. The general idea of time thus developed next becomes differentiated into past and future; finite minds, according to Lotze, on the one hand "assign themselves their position in relation to their more remote or nearer *conditions* as to what is more or less long past"; and on the other hand, "to their more remote or nearer *consequences* as to a future. . . . more or less late" (p. 343).²

Fully adequate criticism of this view would necessitate the formulation of a complete psychological theory of time cognition; but I can only refer to what appear to be its two principal defects. In the first place, our judgments of the past, and of the future, are here regarded as springing from, or based upon, our knowledge of conditions, and of consequences, respectively. But the recognition of conditions as such (and especially of any complex system of conditions) as being "more remote, or nearer," is at all possible only to a highly developed mind, and certainly follows, even if it be not dependent upon, a simpler time cognition; so that Lotze is here explaining a relatively simple mode of knowing by means of a more developed mode—a plainly illogical procedure. Then again if we interpret our time consciousness thus, in terms of conditions and consequences, it must be noted that every "consequence" is really itself a "condition"—is but a condition of a special kind. For Lotze's own fundamental principle that the universe is throughout systematic, plainly implies that *every* phenomenon, without exception, is at once a condition, and itself conditioned; but to assert this and

² Italics mine.

nothing more, would be to leave ourselves without any criterion wherewith to distinguish one condition from another; and when again we do so distinguish, we find that the only distinction we can use, in the first instance at all events,³ is that between "earlier" and "later." It is thus that "conditions" become determined, some as "consequents," some as "antecedents," these terms, not condition and consequence, being the proper antonyms, and a "consequence" (as indeed its etymology should suffice to show) is really nothing but a later condition. So not only does Lotze ground the simpler mode of cognition in the more complex, but further the very terms in which he explains the temporal, themselves imply time, i. e., his theory quite illogically presupposes the very problem which it purports to explain.

c. The same defect is apparent from another point of view. Our relation to conditions, Lotze has said, determines the past, and to consequences, the future. I have tried to show that this explanation fails, because a consequence is itself simply one special type of condition—is a late condition. But secondly, we distinguish further (Lotze continues) not only past from future, but also, within the past itself, the more remote from the less remote; and similarly with the future. The "more remote, or nearer, conditions" correspond to the "more, or less, long past" and "more remote, or nearer, consequences" to the more, or less, late future. Now let us for the moment admit Lotze's distinction between conditions and consequences to be logically sound; even then I think that Lotze is once more explaining time in terms which themselves either presuppose time, or if they do not, then become meaningless for the purpose he has in view. For what is here the meaning of "remote"? I think it can mean only

³ In the higher types of knowledge, this temporal criterion is supplanted by others which are deeper and truer; but whether it is itself ever wholly supplanted by these remains a problem—indeed *the* problem.

(a)
expla
tauto
possi
hims
mean
offer
futu
d
divor
belon
prop
inate
retur
(p. 3
Why
time
deve
mere
succ
be ca
I
strik
later
It is
an a
that
dom
of a
ity a
such
able
such
tent
othe

(α) remote in time—and in that case nothing whatever is explained, for the explanation becomes at once obviously tautologous; or (β) remote in space—and this is an impossible spatial metaphor, condemned as such by Lotze himself (p. 315); and thus neither of the only possible meanings of "remote" actually supports the explanation offered by Lotze of the nature of our ideas of past and future time.

d. But in contrast with this abstract idea of time as divorced from all content, Lotze regards the succession belonging to the operation of things as being "the most proper nature of the real" (p. 354)—we must not eliminate from reality "the lapse of events in time": here Lotze returns "to complete agreement with the ordinary view" (p. 350). And thus it seems to be a legitimate question, Why should this marked contrast hold true? If empty time, the "creation of our presentative intellect" (*ibid.*), developed in the course of our commerce with reality, is merely a "fantastic image," why on the other hand should succession or the lapse of events—i. e., I take it, what may be called concrete or filled time—be itself essentially real?

I do not think that Lotze at all adequately justifies this striking diversity in his views, which (as we shall see later) has an important relation to his treatment of space. It is true that Lotze regards being as "a continuous energy, an activity or function of things" (p. 122), and also further, that while continuity need not necessarily be in itself predominantly temporal—as, e. g., the continuity of a line or of a course of thought—still a continuous *energy* or *activity* appears to be quite inconceivable apart from time. For such an activity must surely have within itself distinguishable phases or moments, and these again, merely as being such moments, and apart from the peculiar concrete content of each, have a common unifying relation to each other. For in the absence of (α) such diverse phases the

activity would be wholly featureless and meaningless; while without (β) such a prevalent relation, it would lack unity; in short, the energy of being must be a unified diversity. It is, then, this indispensable relation between the moments of being—which, while all pervasive, is at the same time wholly and equally indifferent to all specific contents—that constitutes real time or succession; for in its indifference it resembles empty or abstract time, but differs essentially from it in being inseparable from the total content of real being, thus regarded as an activity.

Such a view, again, raises two fresh problems: (1) In this continuous activity of being, what *value*—what *significance*—has time? I. e., why is the temporal relation at all necessary? What is gained by thus conceiving a dynamic, rather than a static reality?—(2) In what sense can such an endless activity truly be said ever to have any *total* content? I. e., how can totality be really harmonized with continuous activity?

Both of these questions find their solution, I think, in the deeper view of reality as being not only a bare activity, but further the development⁴ of a self-realizing whole, which necessarily expresses itself, as its development proceeds, in phases having always higher and higher values. Viewed from this standpoint, the time relation subsists not only between stages which are merely earlier and later—for, merely as such, why need they be distinguished at all?—but between moments which, as they appear, constantly increase in value, so that the later is also the fuller and the richer; while at the same time each moment, summing up within itself all earlier phases, is thus also a real totality, and so something more than merely a fresh aspect which

⁴Development is a richer conception than evolution. In the latter the initial stages are, usually at least, regarded as being much simpler than the later; but this simplicity is really largely illusory, being due to a lack of insight into the true character of these early stages themselves, combined with an illegitimate limitation of the subject-matter apart from the developing whole.

might be severed and held apart from all preceding and all later developments. It is true that this general view of reality assumes each totality, as soon as it is formed, to be absorbed in a later and still richer phase; but this in no way detracts from its nature as being in itself while it exists a total, a summing-up of changes so far as these have gone hitherto. Unless we adopt some such view, the activity or energy of being becomes a mere featureless series of detached phenomena, in which some are first and some last, but all alike without reason, and therefore without meaning or value as a whole.

II.

In considering next Lotze's treatment of the nature of space, it is essential to keep before us his attitude toward time—empty time being unreal, while the lapse of events, or succession, on the other hand is fundamentally real. And since time and space appear at first sight together to constitute a pair of allied categories, it would seem natural to expect that space also, like time, should have a kind of dual nature—have one aspect which is a part of true reality, together with another which is merely formal and unreal; but Lotze, on the contrary, regards space as being, unlike time, *wholly* subjective and phenomenal.

a. But I think it must be admitted at the outset that there exists some obscurity and difficulty here which arises from Lotze's phraseology; for while in dealing with time his language is quite clear and consistent, some degree of ambiguity appears to characterize all his arguments on space.

What is the form of our consciousness of time? Is it a perception, or a conception? I think the question is still debatable; Lotze, however, holds that "we have no primary and proper perception of it at all"; its apparently perceptual character being "only obtained by images which are bor-

rowed from space" (p. 315); and this view is consistently adhered to throughout his entire consideration of time; but in his discussion of space we do not appear to be given the same clear presentation of the issues involved.

Lotze states the problem with which he is concerned to be that of "the metaphysical value of space. . . . we only want to know what kind of reality we are to ascribe to space as we have to picture it, and with what relation to it we are to credit the real things which it appears to put in our way" (p. 231). Now "space as we have to picture it" must mean, I think, "space as we perceive it," for with any other meaning its subjectivity would seem to be presupposed from the outset;—an idea proper, e. g., cannot be "pictured." This view harmonizes with Lotze's own chapter heading, "Of the Subjectivity of Our Perception of Space." But here again a further degree of exactness appears to be necessary; for by "perception," taken literally, we should mean the conscious process so called, and this again without question is subjective; so that there can remain only the object⁵ of this process—i. e., the content perceived, in this instance space—about which we may inquire whether it is subjective or objective.

But to the problem as he has himself thus defined it, Lotze by no means confines his arguments, for he passes on almost immediately from "the impression of a finite extension which is presented to our senses," to the completion of this "picture" in "the idea of infinite space"; and this, rather than perceived space, itself then becomes "the matter whose truth and validity are in question". . . . "We must first try to define what space *as represented in our minds* claims to be" (pp. 232f).⁶ Thus there becomes asso-

⁵ The use of "object" here may be thought again to presuppose the issue; but it is used in the sense in which everything without exception, which can in any way be brought before consciousness, is thereupon an "object" for consciousness.

⁶ Italics mine.

cia
spa
our
con
Lo
pre
wh
and
spa
Th
me
poi
cat

tio
val
cis
for
hel
(V
tio
Lo
nor
"re
pro
it i
tha
cip
but
mo
our
of
sur

men
our

ciated with the problem of the "metaphysical value of space," the further question of the "truth and validity of our idea of infinite space"; but again, there is a further complication, for in the chapter on "Deductions of Space" Lotze refers to space as "only the subjective form of apprehension evolved from the nature of our souls" (p. 276);⁷ while in discussing the bearing of his theory upon things and their relations, he refers to the "active cause of our *spatial idea* in which we picture their locality" (p. 260). This last instance, however, may perhaps be regarded as merely loose phraseology, for in another connection Lotze points out that ideas themselves cannot have spatial predicates at all (Vol. II, p. 251).

Amid this variety of terms, each with its own implications, what is now the precise issue? It appears that the validity of our idea of infinite space has after all no decisive bearing on the question of the subjectivity of space; for merely in itself "infinite extension would not have withheld us from recognizing" the reality of even *empty* space (Vol. I, p. 323). Nor is it easy to reconcile the designation of space as a "subjective form of apprehension" with Lotze's own accounts of our consciousness of spatial phenomena. Perception as a process, according to Lotze, "really does nothing"—there is in fact no "working or process at all as a means to the production of its content; it is nothing but a direct receptivity (*loc. cit.*, p. 275), so that, if space be a "form of apprehension," this same principle must surely apply: it, too, in that case is "nothing but a direct receptivity." But how is it possible to harmonize these expressions with the following description of our experience?—"We all live... under the impression of a finite extension which is *presented to our senses* as surrounding us" (p. 232).

⁷ Cf. also "(Kant) was led to regard (time) equally with space as a merely subjective form of our apprehension" (p. 320); and "spatiality is only our form of apprehension" (Vol. II, p. 248).

How can a form of apprehension—a direct receptivity—be “presented to our senses as surrounding us”? The expression is either meaningless or tautologous. Again, “We presuppose here the ordinary view” (which be it noted is the subject of the inquiry), “the world is extended around us in space; the actions of things⁸. . . produce in perception⁹ a spatial image.” What meaning does this retain if for “space” we substitute a “direct receptivity” which really does nothing—a “form of apprehension”?¹⁰

It is thus impossible to proceed further without some attempt to remove these perplexities as to the exact matter of inquiry; and since this concerns the “subjectivity” of space, it follows (I think) that it must in the first place be taken to be a content of some kind, for the reason that any “form of apprehension” must, as such, obviously be subjective without further argument—all *forms* of consciousness are, as forms and by definition, subjective.

It must again be a *perceived* content—not a content that is only conceived, imagined, or remembered; for each of these again is as such admittedly subjective; and Lotze himself supports this interpretation in his reference to “a direct perception, of time,¹¹ coordinate with that of space” (p. 315). So regarded—as a perceived content—“space” can denote only the “extension which is presented to our senses as surrounding us” (p. 232); and thus the question becomes finally, Is this entity, so defined, subjective or not?

b. But the ground on which Lotze discusses the problem makes possible a still further qualification. For (disregarding the Kantian antinomies) Lotze raises two fresh questions (p. 246): (1) “How can space, such as it is and

⁸ Themselves non-spatial.

⁹ I. e., of course, in “spatial perception” or “perception of space”; the resultant tautology is obvious.

¹⁰ Lotze frequently appears to use loose phraseology. E. g., Vol. II, p. 251, we have “a conscious sensation.” But a *sensation* plainly cannot be itself conscious.

¹¹ Which he does not accept.

must be conceived whether occupied or not, have ascribed to it a reality of its own, in virtue of which it exists before its possible content?¹² (2) How can what we call the existence of things in space be conceived?"

Now it seems to me that the form which Lotze finally gives to the first of these two questions, in itself makes its discussion superfluous; for who desires to assign to space such a reality as would enable it to *exist before any possible content*? As we have seen already is the case with empty time, so likewise empty space, totally divorced from and taken as existing before its content, is merely a formal and abstract conception, and obviously as such subjective. The space whose objectivity actually is in question is the attribute, or property, or constituent of the perceived extended world,¹³ which is perhaps best described as "spatial" rather than as "existing in space." In the perceived physical universe space is one inherent constituent; and if it be proved subjective, all other elements are subjective also—they all stand or fall together. I think we are therefore justified, by the form which Lotze has given to this question, in leaving it without further remark, beyond noting a minor point which appears to involve a self-contradiction. For Lotze adduces, as one argument here, the nature of space as being a "tissue of relations which at starting we represented to ourselves" (p. 247), the truth being, however, that in defining "what space as represented in our minds claims to be" Lotze has said explicitly that "it is neither form, arrangement, nor relation of things, but the peculiar principle essential to the possibility of relations" (p. 234); it cannot, therefore, be a "tissue of relation."

c. We may admit, then, to sum up, that psychology will discover no "image of empty space, formed prior to all

¹² Why "*possible content*"? What criterion have we as to what content is possible or impossible?

¹³ Cf. "How is our position bettered by denying all extension to the real world. . . ." (p. 241).

perceptions, into which the mind had subsequently to transplant its impressions" (p. 260); and as this conception also becomes the ultimate form of Lotze's second fundamental difficulty in accepting the objectivity of space, I think it is needless to dispute his conclusion here that the "assumption that space has independent¹⁴ existence and that things have their being in space" (thus taken as independent)¹⁴ is inconceivable.

For the truth appears to be that Lotze's trenchant arguments are all directed against the ascription of objective reality to the merely formal abstractions—empty time and empty space regarded as existing prior to and independently of all their concrete content; and therefore his objections do not in the least concern the reality of the perceived physical universe which (as Lotze rather curiously expresses it) "our perception sees extended around us" with "clearness and self-evidence" (p. 256); and when the difficulties in Lotze's own positive theory itself are considered, I think it may be asserted that the objective reality of the perceived world may still be regarded as, to say the least, an open question; its time Lotze admits to be real; but its space he does not appear to discuss at all.

III.

Turning, then, to Lotze's own theory of space, we find that he regards "a system of relations between the realities, unspatial, inaccessible to perception, and purely intelligible, as the fact which lies at the root of our spatial perceptions."¹⁵ When these objective relations are translated into the subjective language of our consciousness, each finds its counterpart in one definite spatial image" (p. 263),¹⁶

¹⁴ I. e., in the sense of "preexistent" or "prior."

¹⁵ As Dr. Dawes Hicks points out, this view was held by Plato (*Proc. Arist. Soc.*, Vol. XIV, p. 18).

¹⁶ Cf. "Every particular feature of our spatial perception corresponds to a ground which there is for it in the world of things" (p. 258).

and between this objectively real system, and perceived space, "there exists no resemblance."¹⁷

This conception of reality is one of the fundamentals of Lotze's *Metaphysics*; and it should plainly be the task of his philosophy to attempt some explanation or description of the connection between such a non-spatial real world and that which we perceive as in space. But I think it will be found both that Lotze merely asserts that the ground of this connection is to some degree inexplicable, and that the positive theory which he does propound on this point involves very serious, if not fatal difficulties.

a. In the first place, as regards the manner in which these objective relations are translated into the subjective language of our consciousness, "its explanation," Lotze asserts, "could only be found in the peculiar nature of the soul...but it never will be found; the question is.... unanswerable....All endeavors to derive....this externality....from any possible abstract relation, which are still unspecial, between psychical affections,¹⁸ have led to fallacies" (pp. 273f).¹⁹ We must here "postulate as given the capacity and obligation of the soul to apprehend an unspecial multiplicity²⁰ as in space"; it is "compelled to contemplate these differences not merely as feelings, but owing to a reason in its own nature, as magnitudes of space" (p. 250). This, of course, may turn out to be quite true; still, it is obvious that this appeal to the unexplained "peculiar nature of the soul" implies a serious weakness in the positive arguments for the subjectivity of space.

¹⁷ It may be this lack of resemblance that removes what certainly appears to be a contradiction between the passage just cited and the following: "We do not traverse this perception (of space)...but only the allegation of a being that underlies it, which must be inaccessible to perception" (p. 257);—a "being" which nevertheless (I take it) is still special.

¹⁸ Not here, as elsewhere, between things.

¹⁹ I. e., presumably, a multiplicity of unspecial things; see note 18.

²⁰ "Spatial idea"—see above.

b. But at the same time Lotze does to some degree describe, if not explain, the relation between perceived space and the unspatial real world; and then we find that the translation of "objective relations into the subjective language of our consciousness" becomes an increasingly complex process. The "proximate active cause of our spatial idea," we next find, is not the relations "as merely subsisting between the things, but the concentration in the unity of our consciousness of effects of the things varying in conformity with them" (p. 260). But again, "It is not relations. . . . *between* the things, but only direct reactions which the things are subject to from each other, and experience as inner states of themselves, which constitute the real fact whose perception²¹ we spin out into a semblance of extension" (p. 263); and then finally the soul combines "its occasional particular impressions in the definite situation in space in which they are the image of external objects,"—an activity which, as we have seen, is inexplicable except as being due to the "peculiar nature of the soul"; while, further, these particular impressions, as inner states of our consciousness, are wholly dissimilar to the space in which they are combined (p. 257).

But even if we concede to the soul this "peculiar nature," we must demand in the interest of reason itself, both that such a nature be not self-contradictory, and that it be capable of providing a relatively simple explanation, so far as possible, of our problems.

"What important advantage," we must ask, to use Lotze's own words, is secured "by the assumption of this enigmatic" nature of the soul? I venture to say, none whatever; for Lotze's theory becomes so complicated in its actual detail as to be in the end self-contradictory.

c. In the first place, the world of things is translated

²¹ "Perception" is plainly the wrong term to use here, for Lotze has just asserted that the realities—the things—are "inaccessible to perception."

into spatial *positions*—the soul combines its “particular impressions in the definite situation in space” (p. 274); “within our consciousness the reactions (of) things expand into extension” (p. 279); “each [objective relation] finds its counterpart in one definite spatial image” (p. 263). These three statements are all consistent. But in contrast with these, we have that the soul contemplates these differences “as magnitudes of a space whose parts are beside each other” (p. 250). The inconsistency here is, I think, obvious,—things are said to be cognized first of all under the forms of situation, extension, and spatial image, which are all in the category of pure space, and then, secondly, under the category, plainly quite distinct, of spatial magnitude.

d. Then again, although there exists no resemblance between the intelligible relations and perceived space (p. 263), still both systems are in their diverse ways real (p. 258). Lotze does not question the reality of space—for our perception; but “all spatial determinations are secondary qualities, which the real relations put on for our minds only”; and thus we have a duplication of reality—a duplication, be it noted, not a grading into degrees. The question is surely permissible here, Why should reality double itself thus? What ground is there, either in the nature of reality itself, or (what comes to the same thing) in the “peculiar nature of the soul,” for such a complication by which nothing whatever is gained? For if the soul is able to comprehend the whole most complex reality of space as this is perceived, why may it not comprehend the non-spatial real corresponding thereto, directly and without the intervention of secondary spatial forms? It cannot be because the non-spatial real is as such too complex to be directly apprehended, for “each of (these objective relations) finds its counterpart in *one definite* spatial

image to the exclusion of all others" (p. 263)²²—the degree of complexity is in each case exactly the same. And again, the various properties "of colors and tones," admits Lotze, "are not lost to us because we cannot satisfactorily symbolize them in space" (p. 283); so that what is gained by the substitution of spatial reality for the non-spatial is difficult to conceive.

e. But it is when the actual details of Lotze's theory of the translation of this non-spatial system into space-perception are examined, that its most serious difficulties reveal themselves. Assuming, in the first place, that consciousness or the soul is such that it is able to contemplate a certain type of feelings not merely as feelings, but as magnitudes²³ of a space, we find that Lotze now lays down further conditions. "If a consciousness which recollects its own different acts or states, experiences a number n of impressions of any kind in a succession which it cannot alter at pleasure; if, in the transition from each impression to the next, it experiences alterations, sensibly homogeneous and equal, of its own feeling;²⁴ if it abstracts from the various qualities of the impressions and only calls to mind the form under which they cohered"—all these processes or activities are necessary to create "the picture of an orderly series or system of series, in each of which between the terms $m-1$ and $m+1$ it is impossible for m to be missing" (pp. 250f).

But as a theory of the psychology of our perception of reality under spatial forms this account presents many difficulties. In the first place, the *perception of space itself* is left entirely unaccounted for—is regarded as due to some

²² Italics mine.

²³ See above, regarding magnitudes; but the difficulties remain, whether the theory concerns pure space, or magnitudes of space.

²⁴ It is these differences, in particular, which are contemplated as also magnitudes of space.

undefined "reason in (the soul's) own nature." Then it is further plain, from a careful analysis of the whole passage, that consciousness

α recollects its states,

β under the form of number, and

γ of unalterable succession (and therefore of time);
then between each impression thus apprehended consciousness experiences

δ alterations, which are

ϵ homogeneous and equal.

It is these latter experiences—not the primary impressions, but the alterations accompanying the transitions between these—which are "contemplated not merely as feelings but (also?) as magnitudes of space." But with processes and contents so complex as the theory is thus seen to necessitate, it must surely be impossible that introspection should fail to reveal them under their own proper character. Can we then discover in our experience any concrete states, or definite activities, which, while themselves non-spatial, occur in groups whose succession is fixed, and which are inseparably associated with either spaces or magnitudes of space? In the whole of non-spatial experience psychology can detect no actual content whatever such as that, when we experience the transition from one phase to the next, we thereupon become aware either of space or of spatial magnitude.²⁵ That the whole experience, again, is subconscious is also impossible, for its character—number, unalterable succession, abstraction, but above all *recollection*—are at once too definite and too many in number; so that psychology cannot but pronounce

²⁵ It does not seem to be quite clear whether the equal and homogeneous alterations in feeling are experienced as feelings, and *also* as spatial magnitudes, or merely as magnitudes alone. The first alternative would plainly increase the psychological difficulties of the theory; but it seems to be implied by the argument on page 260.

the phenomena which are obviously necessitated by the theory to be non-existent in reality.²⁶

f. Analogous difficulties which are equally serious attend the alternative exposition of this same theory which is given on page 260:²⁷ the "*series* of feelings of homogeneous change, experienced in the transition from the impression *p* to the impression *q* is felt as the distance *pq*."²⁸ Here, in the first place, a *series* is said to be felt as a simultaneity—as the distance *pq*; but the view which Lotze has propounded as to the reality of succession—of the lapse of events—makes this type of apprehension wholly impossible, for it is the apprehension of a succession, which Lotze takes to be known as real, under the subjective unreal form of simultaneity in space. Further, the essential nature of any series is such that in its apprehension as a series some of the terms or phases must be less vivid than others—must, to be apprehended at all, be remembered; but in the perception of a distance *pq*, all the elements therein—*p*, *q*, and *pq*—must be equally vivid; and thus a *serial* content, whose terms must differ in intensity of feeling, is here said to be apprehended as a spatial content, whose constituents are equal in intensity throughout.

But the most serious difficulty, which, indeed, appears to vitiate Lotze's entire theory, is that the apprehension of *distance* must necessarily presuppose the consciousness of space; the apprehension of spatial magnitude must imply the prior apprehension of pure space; and therefore even were we to admit that certain series of impressions can be

²⁶ Lotze admits that these "reflections are utterly foreign to the common consciousness, (which) never has an inkling of the intermediate processes required to produce feelings" (p. 259). But this does not help his theory, which cites not the processes which produce feelings, but the *feelings themselves*; and of these reflective consciousness at least must have some cognizance.

²⁷ Slightly abridged; italics mine.

²⁸ In the use of *p*, *q*, and *pq*, there appears to be some degree of metaphor; for the "distance *pq*" cannot really subsist between impressions *p* and *q*, unless *p* and *q* are already points in space; here the meaning of *p* and of *q* taken separately, is not the same as their meaning in *pq*.

thus apprehended under the form of distance, still to make this possible we should be forced to concede or presume the prior and in itself unexplained consciousness of pure space or extension. If, i. e., our consciousness of reals is (as Lotze asserts) possible only under the form of distance (or other spatial magnitude), then to what does the necessarily prior apprehension of pure space or extension correspond? Not to yet another real, for reality is one; but if not, then the perception of pure space corresponds to nothing in reality and becomes altogether gratuitous and inexplicable.

g. But that our consciousness of reals takes the form not of space itself, but of distance in space, is repeated in another important passage: "The meeting of these two actions (i. e., of the reals P and Q) in our consciousness causes. . . the possibility of a comparison and reciprocal reference of the two; secondly. . . the necessity that the result of this comparison should assume the form of distance in space to our perception; and finally, the magnitude of the difference felt between the two actions determines. . . the visual angle by which we separate the impressions of the two elements (P and Q)" (p. 264); and as we found was the case with the account already referred to (p. 250), so here Lotze describes a very complex conscious activity, and one which, again, cannot be regarded as occurring subconsciously, because the visual angle is stated explicitly to depend on the "difference *felt* between the two actions" of P and Q.

But this complexity of the conscious processes involved has further implications. That space is purely phenomenal is, as Lotze himself asserts, an assumption—not a "direct and overwhelming impression" (p. 259); and that reality is a "system of relations, unspatial, inaccessible to perception, and purely intelligible" (p. 263), is a conclusion from "reflections on the true state of the facts" (p. 259).

Thus, our knowledge of reality follows, through reflection, on our naive but erroneous belief that space is itself an ultimate real. But in the passage just cited this postulated sequence of reflective consciousness upon our naive belief is completely abandoned, and our perception of distance is there presented as being itself the result of processes of reference and of comparison which are themselves plainly only possible at all at a high level of consciousness; so that as the consequence of these complex activities²⁹ we obtain (α) our naive, direct, overwhelming, but deceptive perception of space, and then (β) pursuing "reflections propounded" by the assumption that space is after all phenomenal, we attain our knowledge of unspatial, imperceptible, intelligible reality. Such is the final form of Lotze's theory of space and reality. Again it must be asked, Does introspection reveal these felt actions, comparison and reciprocal reference, underlying our perception of space and distance? Too complex to occur subconsciously, they are also impossible during earliest infancy; while the spatial judgments of some of the lower animals are instinctively highly developed almost from birth, so that there is in their case no time whatever for the occurrence of the supposed comparison and reference.³⁰

IV.

But in spite of Lotze's insistence on the phenomenal reality of space, and on the non-spatial character of reality, he adopts throughout his consideration of "The Formation of Our Ideas of Space" the every-day attitude of naive realism: "We presuppose the ordinary view—the world

²⁹ For both comparison and "reciprocal reference" are impossible without some imagery or ideal content.

³⁰ These, however, must not be confused with two incontestable groups of phenomena: (*a*) the gradual formation of our developed spatial judgments; these, however, are founded on perceptions themselves vague and indefinite, yet still spatial; (*b*) "the manifold intermediate processes (both physiological and psychical) required to produce" our developed judgments; but of these we are never conscious—they are not "felt actions."

is extended around us in space; we and the things in it have determinate places in it; actions are propagated. . . . and produce a spatial image; the component parts of which have the same reciprocal positions as the external things by which they, as sensations, were produced" (Vol. II, p. 248). Although I think that this method of treating the problem is scarcely legitimate, still it is impossible to consider Lotze's psychological theory on any lines other than those which he himself chose to adopt. At the same time I do not think it necessary to do more than point out a few obscurities in Lotze's treatment of the subject.

a. The spatial image produced in perception, according to Lotze, consists "in a number of excitations of nervous points" (Vol. II, p. 249); and the question thus arises, "How this fact of nerve-excitation becomes an object of knowledge for the soul." But, surely, this is a radical misstatement of the real facts of perception. Each spatial image is certainly connected in some way with excitations of nervous points; but the image is not these excitations. The nervous processes themselves are imperceptible molecular motions, which never appear in perception at all, whereas the spatial image has form and color and is in perception. Lotze has here plainly confused the nervous changes which are conceived by science, with the actual image perceived by means of the eye; and vision in itself has little to do with making nerve excitations "an object of knowledge." That is the work of physiology and has nothing to do with vision as such.

b. Lotze next conceives these excitations as undergoing "the transition to consciousness" (Vol. II, p. 252);³¹ and since consciousness is non-spatial their spatial character must now be dispensed with, and so "the continuation of the process consists. . . . in the production of an idea—the

³¹ The obscurity is increased by the equivalence here of the terms "excitations" and "impressions."

idea of space....From this non-spatial material the soul has to re-create entirely afresh the spatial image that has disappeared"; and this it does "for unknown reasons." It would certainly be difficult to find any others to justify the general incomprehensibility of this entire process. For having first of all confusedly identified the spatial image (itself distinct from the external spatial thing) with the nerve-excitations, and having then, simply because consciousness itself is non-spatial, dispensed with spatiality, we find that the soul re-creates both the *idea* of a space, and the spatial *image* that has once existed and has disappeared. But this final spatial image cannot itself be an idea, nor can it be identical with nervous excitations, and the resultant obscurity seems to be Lotze's penalty of having abandoned, merely for the convenience of argument, his initial fundamental principle of the non-spatiality of the real. We appear, then, to have little beyond pure guesswork, rather than any philosophic assignment of relevant conditions; one attribute is added to another, suppositiously and "for unknown reasons," thus recalling the older method of "preestablished harmony"; forgetting that it is always the essential task of philosophy to construct its own harmony.

J. E. TURNER,

LIVERPOOL, ENGLAND.

use
tha
oph
per
exp
be
unr
can
sen
and
By
is
rea
onl
as
rea
In
us
Th
is
tru
not
pos
the
an

Mar

CRITICISMS AND DISCUSSIONS.

TIME AND SPECULATIVE PHILOSOPHY.¹

In this paper I shall use the term "speculative philosophy"—used by Prof. Bosanquet to distinguish his type of idealism from that of the "mentalists" and "panpsychists"—to mean that philosophy which holds that "the view of the universe as a uniform and permanent system of natural laws is a necessary element in a rational experience."² This philosophy cannot "conceive how there could be a rational life without an apprehension of an objective order; unmoved by our clamor; indifferent to our moods; with which we can hold commerce only on nature's own terms." In fact representatives of this type of idealism do not distinguish between reality and truth. What is real must be true and what is true must be real. By "real" they mean systematic coherence, organic unity: a thing is only real when we consider it in its relation to the whole of reality. But there are degrees of reality: we may say a thing is only phenomenal when considered in isolation, when considered as an independent existence it approaches more and more perfect reality as we understand it in its relation to the rest of existence. In other words, the reality of an important historical event, or let us say a chair, is real in proportion to its organic structure or unity. The same thing is true of our concepts or judgments: a judgment is false if it is out of relation, it becomes true and its degree of truth is determined by its organic interrelation to other judgments, not only in the present universe of discourse but for all actual and possible ones. So we may say that only that judgment which implies the whole universe is true in the highest degree. The reason why an esthetic experience is more representative of the Absolute than

¹ Delivered before the Philosophical Conference, Columbia University, March 18, 1918.

² Creighton, "Two Types of Idealism," *Journ. of Philos.*, 1917, p. 534.

let us say a chair, is because in the esthetic experience there is more harmony, organization, interdependence, and organic unity.

I do not question that to be able to live at all we must live in a rational system and that the criterion of truth is coherence, or rationality. But I think we may question whether it is necessary to have an absolutely coherent system of the type just described in order for us to be able to transact business, or for me to be able to communicate the thesis of this paper. I think that we are forced to admit that a judgment or an act of experience is more intelligible when seen in its relation to other judgments or acts of experience, and that the *ideal* of knowledge would be to see each and every judgment and each and every act of experience in its relations—if the relations would be static and eternal—to all and every other judgment and acts of experience. But speculative philosophy goes farther than that, it assumes that what I call *ideal* in my criterion of truth is really an already existing reality. In fact, in this view that is all that reality is, i. e., an organic interdependent unity. The unity is so complete that to speak of terms, or relations, or time, or a subject as over against an object, is to speak in terms of contradiction.

The leaders of speculative philosophy claim that a theory of reality which holds that reality is a unity, must also hold that this unity is a timeless and changeless unity. It is true that some are ambiguous on this point, as, e. g., Hegel in his relation of logic to the philosophy of nature, but most of them are very clear. If we ask them why it is that we apparently live in a world which endures in time, changes and develops, there is no answer. Or we may say, suppose you are right in showing that a rational analysis of experience leads to a conception of a static and timeless Absolute, it is still your business to show, why we should have this world of appearance, and, furthermore, what really is the relation between these two worlds. To that—if we take Mr. F. H. Bradley as their spokesman—the answer would be, we do not know, all we can say is that it is so, and the rest is a mystery.

The absurd contradictions which the analysis of time and development leads to, forces the speculative philosophers to the conception that time and development are only illusory. The best illustration of that we find in Mr. Bradley's book *Appearance and Reality*, in which he employs "Zeno's method" to prove his conclusions. He finds that the concept of continuity in the principle of causality involves empty space between the cause and effect, and also that the

concept of causation leads us into infinite regress; furthermore, the fact that we arbitrarily select the cause of an event from a plurality of conditions leads him to suppose that the concept of causation is only an appearance and not a reality. But it never enters his mind to ask the question: if reality is the organic unity of appearance, does not change in phenomenon affect it? In the final analysis the speculative philosopher must accept one of the two following possibilities: first, that the causal world does affect the real world, and the real world consequently cannot be timeless; or, second, that the world which we know, the world which we experience, the world in which causality is real, is an illusion, and after all the real world remains timeless and perfect.

We also find that speculative philosophy goes out of the bound of experience and scientific logic, in its treatment of the law of identity-in-difference as a fundamental axiom of logic. It argues from quantitative and qualitative difference to identity of reality. But I may suggest that all we need in our logic of science is a postulate which holds that the world of qualitative and quantitative divergence falls into *approximate types or groups and subgroups*. In other words: that the world of discrete and divergent objects approximate the *ideal* regularities of type necessary in the logic of science. To put it in still other terminology we may state: that the postulated law of similarity or even of identity of types is only a *limit*—using the term limit in the mathematical sense. I think that we all agree that to comprehend the ground of an event, a person, a book, is to have an inkling of the system of connection in which that event, person, or book takes place, lives, or exists, and furthermore I may suggest that the understanding of the universe involves at least a presupposition that the universe is approximately an ordered and connected continuum. But to be able to admit this, does not require the admission that the relations and behavior of each and every bit of experience is definitely and precisely settled forever. It only means that every bit of experience acts and develops within the limits of the type or class to which it belongs. But in approaching the limit there are infinite possibilities of relations, e. g., that there can be an infinite number of circles drawn for each and every point between any two points of a compact series, no matter how infinitesimal the distance between them may be. Then I may say: that knowledge comprehends the approximate order and interconnection of reality, but as distinct and unique phases of real entities. And finally, the principle of continuity means that

we should analyze the complex as far as it is possible, keeping in mind that there is discontinuity, e. g., qualitative changes. In physics we may approach quantitative continuity in the transformation of energy, but not in qualitative changes. Or in psychology we may speak of quantitative continuity of nerve and muscle reaction, but we cannot speak of continuity of the "feeling-tone" accompanying this reaction. Consequently, if we postulate absolute continuity we are really ignoring certain discrete phases of change and development, and our postulate is illegitimate and false, and furthermore, as a postulate for scientific logic, unnecessary.

Now let us examine the concept of teleology in speculative philosophy. If the universe is an absolutely perfect organic unity, changeless, timeless, in which there is always a perfect equilibrium, how can we speak of the world in teleological terms? The absolutist says we cannot. The universe as a whole is not teleological, teleology holds true only of the world of appearance. It is true they say that teleology is a higher concept or is of a higher order than the concept of causality, but nevertheless teleology does not pertain of the Absolute because teleology involves time and novelty. Bosanquet says teleology is a subform of individuality, while the Absolute is not a self, because a self presupposes a not-self, says Bradley. Consequently selfhood or personality is an appearance.

We find Bosanquet arguing for the conservation of value, but he seems to forget that the conservation of value is meaningless unless the values are conserved for individuals, but again the individual as a personality is only transitory and illusory. It is true that human beings have a higher degree of reality than plants, and plants again a higher degree of organization or reality than stones, but nevertheless in the final analysis the difference is only in degree and not in kind. And furthermore does it follow that individual things are illusory, because we have to assume a medium of interaction. Do we have to assume a ground of interaction for the beings that interact? And suppose we admit that there is such a medium, or ground, does it follow that individuals are only differentiations or negative differentials of that medium? It is true that individuals communicate with one another, consequently there must be some sort of physical medium of interaction. But this medium is only a tool of ours to enable us to carry out our communications, but the beings who communicate or interact do not have to be mere differentiations of that medium.

que
asso
the
of t
Heg
zati
fere
form
natu
of t
clain
the
total
into
relat
and
cruc
Bosa
indiv
as a
indiv
othe
furl
they
vidu
mean
is no
We
iden
they
all in
whic
vidu

the u
it ca
and

3
matic
Libra

Spinoza and especially his followers Hegel, Bradley, Bosanquet are right in saying that a human being develops only in his associations with other selves and their institutions. The goal of the individual is the complete going on to realize more of the unity of the Absolute, to come in more intimate union with Him, says Hegel. The individual becomes more perfect and free in his realization of necessity and its place in God, says Spinoza. The difference between Hegel, Bradley, Bosanquet, and Spinoza is that the former, i. e., Hegel, Bradley, and Bosanquet, speak of the personal nature of man as spirit, while the latter speaks of him as a mode of the attribute of substance. As I have said, they are right in claiming that the individual develops by individual transcendence,—the individual who lives by himself, who thinks that he is the sum total of the universe, is an individual who shrinks more and more into nothingness. The individual finds himself, so to speak, in his relations with other individuals, e. g., his friends, family, social and political institutions, by living in general relations. But the crucial question which Spinoza and especially Hegel, Bradley, and Bosanquet avoid, is whether the individual through realizing his individuality in his relationships with others, loses his individuality as a finite center of experience in the Absolute or God. If the individual grows in proportion to his intimate relationships to others, then we must also admit that the Absolute grows. And, furthermore, if finite individuals are literally parts of an Absolute they are not finite individual centers of experience, but if the individual is not identified with the Absolute, but by "harmony" is meant just a relationship between two terms, then the Absolute is not a perfect unity but a synthetic relationship of individuals.... We may say still further, that if Hegel, Bradley, and Bosanquet identify literally individual elements with the unity of the Absolute they must say with Spinoza that all value—and with that all desires, all interests, all purposive activity whether conscious or unconscious which value involves—is lost and destroyed with the loss of individuality in the Absolute.³

Speculative philosophy also claims that progress takes place in the universe and in the individual, but not in the Absolute. In fact it cannot take place in the Absolute, because the Absolute is perfect and timeless. But how is it possible? we may ask. How can the

³ As I have pointed out in another connection ("The Influence of Mathematical Concept on the Structure of Spinoza's System," Ohio State University Library, 1917).

world of individuals, of philosophy, of history, of science, etc., develop and change and the Absolute which contains them be changeless? They say progress or development in general is only—as concerns the Absolute—a constant redistribution of parts within the whole, that is, one part of the Absolute may progress but the other must retrogress, in order that the unity may be in perfect equilibrium. But even that, it seems to me, involves time. And furthermore, did not progress—that is, the development of Spirit—in Hegel's philosophy play an important role? Using Hegel as an example we may ask: would the speculative philosopher be willing to say that, e. g., Hegel's analysis of the development of monarchy from a feudal state—in his philosophy of history—characterizes a decisive progress in the universe as a whole? But he answers in the negative, the Absolute never progresses. Consequently progress is also an illusion. But I may suggest that experience teaches us differently, and our logic of science—as I have mentioned before—does not require static ideal relationships. Therefore we are justified in saying that the Absolute of speculative philosophy is not a teleological or progressive unity, it is timeless; not being teleological it must be mechanical. Speaking in the strictest terms it cannot even be mechanical, because mechanism implies the principle of causality and that implies time.

* * *

I think it will be profitable to mention briefly the philosophic doctrine that appeared recently in the *Journal of Philosophy, Psychology and Scientific Methods* under the name of "Structure,"⁴ in so far as it has any bearing upon our general problems.

The main thesis of this article is the same as the fundamental postulate of speculative philosophy. "The world is a rational, orderly world, independent of man's thinking or activity." The "thisness" in virtue of which the world is rational or logical, the author calls "structure"—it is "that which persists through all changes, or that which is defined in its own terms, or that which stands under, or that which has attributes, or that the knowledge of which is conclusive and final....in fact it means something absolute without meaning God,"⁵ it is eternal and timeless. But there is a decisive difference between "structure" and the Absolute. Structure does not imply an all-inclusive universe of which every-

⁴ Vol. XIV, 1917, p. 680.

⁵ F. J. E. Woodbridge, *loc. cit.*, pp. 686f.

thing that exists or can be conceived is a part. Outside of structure there is flux, activity, although not independent of it, nevertheless not identical with it. Structure is timeless, static, and eternal, but the growing residue or flux is "subject to this structure suspended in time fully as much as the displacement of its parts is subject to a structure extended in space."⁶

With such conception of the universe the author of this article has the advantage over the speculative philosopher in that his theory of the cosmos has room for the postulates of causality and continuity, and even the postulate of teleology, all of which are so essential in a logic of science. For him change is a genuine reality, although change must follow definite laid-down forms or laws which are eternal and changeless in themselves.

But if we look closely into this type of cosmos and examine the change that takes place, we will find that it falls short of the change which seems to me to be characteristic of nature. The change which is possible in a world in which structure is eternal and supreme, is a change which must follow definite, inherent, unchangeable, and perfectly organized laws. There is no genuine novelty possible, but only in so far as there is a redistribution of the residue, the same matter takes up new but eternal forms. However, change and development appears to be of somewhat different character. It seems to me that we have plenty of examples where genuine novelty is developed through men's activity and ingenuity, novelty which may follow general types of development, but which also produces new organization and values which are not inherent parts of the movement, that is, not the necessary consequences of that movement, so that there is genuine spontaneity. The development of new qualities are the functions of changing human needs and not of rigid structural laws, although we must admit that the realization of these values is limited by their proximate limitations to which matter is subject. I may suggest that individuals are plastic and capable of developing new relationships, as a result of their development. The order and organization of individuals, e. g., in a society, is an order which is the result of the conduct and behavior of its individual members, and none of it existed even potentially in the structure as a presupposition of such a conduct; all we can say is that lower forms of organization have developed into higher forms of organization.

Or again in human creation we have plenty of examples of

⁶ *Ibid.*, p. 682.

genuine novelty. Take, e. g., a new picture, a new book, a new philosophy, a new symphony, or even a new type of printing-machine or flying-machine, in each case we may have the same energy or mass, there may not have been any new energy created, but surely new values, new *purposive* energy has been created. A picture is more than a distribution of paints. Change for the most part is "creative synthesis," a concept developed by Lotze and baptized by Wundt.

Now let us examine the concept of structure itself and see if that is of the type we really find in nature. The author says that "structure is a *discovery and not a hypothesis*. . . . It is met in action and practice by setting one stone upon another. . . ." The laws of thermodynamics and mechanics, the laws of biology and physiology, or the scientific laws in general are the exemplifications of structure as we find them in nature, while we have ventured to suggest that the scientific laws are only *ideal limits* or statistical averages of well-defined types or classes, according to which the elements which are qualitatively different and quantitatively discrete change and develop. Scientific laws deal with abstract characteristics or attributes of things, that is, with motion, or extension, weight, etc., as isolated properties of particular objects and not with concrete objects in their complex relations. In fact to find a "law of nature" we have to abstract from experience, leave the ground of reality, the region of concrete things, and find the functional relations between abstract isolated attributes of particular things.

If A changes at the same rate as B we say that A is a function of B and calculate the derivative which will represent the rate of change. We say that this derivative represents the law or rate of change of the body, or in other words, that this derivative stands for the relationship which exists between the two bodies A and B. But if we carefully observe the change of A and B, whether it be in a physical, chemical, or psychological laboratory, and plot the curve representing this functional change, we find that the relationship between A and B is not as absolute as the principal concept of calculus has it at all, in fact the change plotted does not at all represent a curve. What we really have between our *x* and *y* axes, is a number of points widely distributed, but yet within certain limits representing a continuously progressing curve. We are then told that the variation in the curve is due to defective instruments or to imperfect conditions,—there are so many other factors which play

⁷ *Ibid.*, p. 683.

a definite role and are responsible for that variation. As the claim goes: it is not that the laws do not perfectly operate, but that there are a number of other laws which are operating simultaneously, and the result is this variation. Now that may be so, *but that is a hypothesis and a very useful hypothesis*—just as much as the hypothesis that the sun will rise to-morrow—*but not a discovered fact of experience*. Consequently I may suggest again that the postulate of continuity means only that we ought to analyze regularity in nature as far as we can, but not substitute a timeless, eternal structure or Absolute as the ground of change. The same we may say is true with the postulate of coherence which I have suggested at the beginning of this paper, as being useful and legitimate in so far as it represents an *ideal limit* toward which the actual progressive organization of experience leads; e. g., in the development of economical corporations, military organizations, religious and moral institutions, where there is more and more dependence of each and every individual upon each and every other individual. In a pluralistic world there is an actual striving toward that *ideal limit*. We select and adjust ourselves as we think it is best to move forward toward a progressive organization.

The fundamental postulate of science is that there is a sufficient reason for every happening, that is, every effect has a cause, even though there are no two effects or causes alike. But this postulate is only legitimately used in so far as by causation we mean an *ideal limit* of a contingency table which represents a classification of the degree of dependency between A and B, whether the dependency be qualitative or quantitative. The use of this definition of causation is that it puts variation as the fundamental factor in experience. We may also add that speculative philosophy as a rule confuses the postulate of causality—which, as I have said, is a legitimate postulate of science—with the category of ground and consequence. The first is an appropriate postulate in a finite and temporal cosmos, while the second implies eternal, infinite, and timeless unity. I may again repeat that the postulate of causality with its implications of continuity and uniformity is only used in science for its instrumental value and as an *ideal limit* of what really—as far as we know—is discrete and discontinuous in experience.

The distinctive feature of a pluralistic theory of the cosmos as over against a speculative-philosophy theory of the universe is that in a pluralistic cosmos things are connected with other things

in many ways, but no single thing nor the totality of things includes everything or dominates over everything. The pluralistic cosmos is more like an individualistic society, without any Over-Individual, becoming more and more organized, while speculative philosophy claims that if we deal with the reality of realities, "everything is present to everything else in one vast instantaneous co-implicated completeness" (James).

Pluralism claims that there is nothing in life which is not complex, and that any one of its relations is only one aspect of it, or one way of looking at it. When it acts in that relation or connection it does not imply that it must act in all possible and actual relations simultaneously. For the pluralist things may act one way at one time and another way at another time, but, nevertheless, we still have a coherent cosmos in which each and every individual is a member and each and every individual builds the society: a coherent cosmos but not an organic interdependent unity.

It is in a pluralistic universe that teleological development is possible, that personality with its struggle toward the ideal is intelligible.

I may conclude suggesting a pluralistic theory instead of a monistic theory, not in order to solve the problems but rather to throw light upon the old problems and to create new ones.

A. J. SCHNEEWEISS.

COLUMBIA UNIVERSITY.

THOUGHTS SUGGESTED BY PROFESSOR PATTEN'S RECENT ARTICLE.

Not for a long time have I come upon so stimulating and deep-going an article as Professor Patten's in the April *Monist*, and I desire to set down some of the thoughts that have been suggested by it. My chief interest now is in the forming of some tenable conception of the elementary nature of things, and it is help in this direction that particularly attracts my attention. Professor Patten's view of deficit and surplus mechanisms and of the latter as relatively unspecialized (and in this connection the view of the contrasted functions of the nerves and the blood), his interpretation of dreams, the new angle from which he regards the problem of

the
me;
mean
and
when
prop
stagn
fesse
be m
mar
must
world

follow
artic
pass
woul
is h
tinue
reap
self
"The
arms
striv
of st
227)
plus
(p. 2
at le
move
mani
psych
cann
ical s
I
langu
funda
wish
their
tion,

the transmission of acquired characters—all these greatly interest me; but it is the underlying conception of energy and its ultimate meaning, the view of wishes and their efficiency, their modifying and even creative role in the world of living things—it is these ideas, whether as formally stated, or as implied or at least suggested, that I propose specially to consider. A kind of metaphysic, or the incipient stages of one, seems to me to be presented. This may not be Professor Patten's chief interest in writing the article, but it happens to be mine in considering it. Of one of his special contentions he remarks that they may be regarded as "mere theory," and I suppose I must admit that it is a mere theory that I am interested in tentatively working out here.

The starting-point is expressions of Professor Patten's like the following (for the context I must refer the reader to the original article): "...consciousness [in the simplest organisms] is a pulse passing from point to point on a surface. Its physical manifestation would be a temporary arm like the projections of an ameba. It is here, there, and everywhere in turn but nowhere has it continuity. The pulse rises at a given point and then sinks back to reappear at some other point. Each rising wave is an embryonic self quickly to be replaced by another, equally fugitive" (p. 226); "There is thus a continuity in the series [of successive forms or arms in which the wish appears] in the sense that a single wish is striving for fulfilment, but objectively considered they are a series of surface projections created by a passing pulse of energy" (p. 227); wishes are formally spoken of as "*on their physical side surplus discharges*" (p. 229—italics are mine) or "pulses of energy" (p. 233). The thought seems to be that the pulses of energy (or at least of surplus energy) are in their *inner being* wishes, the movements and the forms the movements take being simply the *manifestation* of the wishes. In other words, it is something of a psychical nature (I use the word in a loose, vague sense, but I cannot think of a substitute for it) that is fundamental, the physical side of it being derived or at least secondary.

I admit, however, that Professor Patten occasionally uses other language which looks as if the physical were regarded as the more fundamental. For example, "*on its conscious side* a pulse is a wish seeking fulfilment" (p. 227—cf. "pulses of energy are on their conscious side wishes," p. 232). More positively in this direction, fear and desire are spoken of as "*representatives* in conscious-

ness" of (respectively) the nerves and the blood (p. 229—italics in both cases mine). And at one point in his discussion he distinctly says that his view differs from that of the Freudians in tracing conscious phenomena to their physical background instead of referring them to a mysterious subconsciousness—"The content of consciousness is a reflection of the physical forces which underly it" (p. 230).

Still other language may be interpreted according to either view, e. g., "the wish which is a pulse of surplus energy" (p. 229), "the pulse rising in consciousness as a wish" (p. 227), "the pulse which is wish in consciousness" (p. 232).

How Professor Patten would finally define himself on the matter, I will not undertake to say—a thinker must be his own interpreter. But I may suggest a view that would measurably reconcile his varying utterances and make a tolerably consistent doctrine—he may assent to it and may not.

1. The fact may be that both ways of speaking are allowable. We may with equal propriety say "Wishes on their physical side are pulses of energy" and "Pulses of energy are on their conscious side wishes," the difference simply being in what we happen to start out by speaking of. In the one case wishes are in the foreground of our attention, and we proceed to say something about them, in the other pulses of energy are first in mind, and we proceed to say something about *them*. The wishes are what we (by "we" I mean now living organisms in general) *feel* or directly experience, the pulses of energy (understanding here outward movements) are what an onlooker (or for that matter we ourselves) might *observe*. Both ways of speaking might accordingly be unified in the following: What we directly experience or feel *manifests* itself to another (or conceivably to ourselves, by means of our outer senses) as a pulse or pulses of energy, i. e., some kind of movement that is visible and tangible. There are not two things, wishes *and* energy—but wishes are energy when viewed from the outside. Wishes are the reality, movements the manifestation. The manifestation is in one sense, indeed, as real as the original thing; only it is derived, secondary, not ultimate; the manifestation might cease (and certainly would, if there were no outside observer and we ourselves deprived of our outer senses) and the original none the less exist, while if the original ceased to exist, the manifestation would *ipso facto* vanish, whether outside observers, not to say ourselves, were on hand or not.

The difficulty of taking this as Professor Patten's view is in reconciling it with what he says of fear and desire as "representatives" in consciousness of the nerves and the blood, and of tracing conscious phenomena to their physical background—language naturally suggesting that the "physical" is taken as original and self-subsistent. I do not know whether it is sufficient to say (somewhat as in the preceding paragraph) that from one point of view fear and desire may "represent" the nerves and blood, while from another the nerves and blood may represent fear and desire; but this is about all I can say. But as to tracing conscious phenomena to their physical background, a word should be said as to just what Professor Patten has in mind in so speaking. He adds at once, "To me it is axiomatic that the content of consciousness is a reflection of the physical forces which underly it. No explanation is satisfactory which does not relate the content of consciousness to these antecedents" (p. 230). It is to be noted that it is the content of consciousness that Professor Patten is here referring to, and content of consciousness is not just the same as consciousness itself. The content of consciousness is what is *in* consciousness or is the object of consciousness, the *definite things* a conscious being is conscious about. Now the *content* of consciousness may well be made up (at least in the connection in which Professor Patten is speaking) of what physical forces give us, without thereby prejudging as to whether consciousness itself is a physical force or an outcome of such forces, or whether physical forces may not themselves be regarded as at bottom psychical in nature, after the manner of thinking of the preceding paragraph. Again, I do not know whether Professor Patten would agree to such a resolution of the difficulty.

2. In the theory I have advanced I may have enlarged somewhat beyond the field which Professor Patten particularly has in mind. His special subject is the wish, which he assimilates, or, shall I say: identifies, with *surplus* energy. However, he speaks of deficit energy (energy at the deficit pole of an organism) as well, and to him this energy has its psychical counterpart in fear—something conservative of energy as wish and desire expend or waste it. But fear, though contrasted with wish, is still something of the psychical order. Fear or wish make up in his view, as I gather, the determining impulses in an organic being—they shape, or shall I say: *are* the inner reality of, all its varied movements—this being as

true of the ameba as of the animal or man. The primacy of the psychical seems so far to be maintained.

Particularly interesting in this connection is what he says of structure. The *structure* of a living being is commonly regarded as something independent of the forces that operate in it or through it. Relatively speaking, it is so, and Professor Patten uses language to this effect. Wishes on the physical side are pulses of energy and not structure, he says (p. 233). He distinguishes between energy and the mechanism on or through which it acts (p. 225). But he also urges that changes of structure may arise and that these may come from the working or pressure of energy. Instead of being an ultimate, nowise the result of more primitive forces, he speaks of structure as caused by energy pulses (p. 231). He even says, "The wish creates structure" (p. 234). For all this, structure may be of different texture or material from energy, and language repeatedly used seems to sanction such a conception. For instance, he speaks of wish as becoming persistent only through structural aid, of its becoming "will" (defined as persistent wish) when "aided" by structure, of a primitive wish (as in an ameba) dying because without "mechanism to aid fulfilment" (p. 227). "The structure is our means of persistence," he says again, "the biological embodiment of primitive wishes" (p. 229). "Structure stabilizes energy, makes it move in particular directions" (p. 223). Language of this sort seems to imply that energy and structure are two separate things, however closely connected or reciprocally influencing one another.

Is there then an irreducible dualism in Professor Patten's conception at this point? I am unable to give an assured answer to the question and can only venture on a little reasoning. As already explained, he does distinctly assert the modifiability of structure. "Wishes modify structure" (p. 236). In advancing a view as to the way in which modifications of structure may be passed on to the next generation—a view very different from the ordinary one as to the transmission of acquired characters and which may almost be called revolutionary—this very clearly appears; the view is the most striking and daring contribution to modern speculative biology that I am acquainted with. According to it, inherited wishes are the central factor in inherited structure. "If wish can modify structure the difficulties of the theory of acquired characters can be avoided. Wishes modify structure, but the new structure has no

pow
not
and
wish
it is
whe
brin
stru
crea
I m
said
thin
stru
wha
bott
psy
preh
at l
if th
the
says
bodi
thin
an e
of v
natu
ethe
and
obje
to in

appr
dari
add
Sch

power to modify germ cell structure. It is the transmitted wish, not particles of the new structure, which modify germ cell activity and hence modify germ structure" (p. 236). In other words, the wish has a dynamic character extending to the very seats of life—it is, indeed, the dynamic agent; and perhaps this is what we mean when we speak of it as energy at all—the more material term but brings the real nature of the wish to light. But if wish may modify structure (whether in the soma or in the germ cells), may so far create new structure, why may it not create structure *ab initio*—I mean, what is the theoretical difficulty in the way? It may be said that wish (energy) must in the nature of the case have something to act on—if not structure, then the material out of which structure is shaped. This may be granted and we may still ask: what is it necessary to suppose that the material is? Down at bottom, may it not be other energy? Or, to use the corresponding psychical terms, may not wish act on wish—or, speaking more comprehensively, fear and wish act on fear and wish? Pluralism, or at least dualism, is inevitable to some extent—there is no shaping if there is not something to be shaped as well as the shaper—but the difference need not be one of *kind*. Professor Patten once says, "The structure is our means of persistence, the biological embodiment of primitive wishes. The will is not an etherial something, but substantial bodily structure. It is wish objectified, not an entity dematerialized" (p. 229). Why may not this conception of will or wish be extended so as to cover what is of psychical nature in general, and this be held to be in its entire range no etherial something, no entity dematerialized, but the very pulse and life, the inner being, of matter itself—matter being but its objectification, its appearance or manifestation to anything related to it or observing it from without?

A view going so far as this may not have Professor Patten's approval, and it is admittedly tentative and theoretical, but his own daring suggestions embolden me to indulge in it here—and I may add that it is closely akin to the view of two notable thinkers, Schopenhauer and Nietzsche.

WM. M. SALTER.

SILVER LAKE, NEW HAMPSHIRE.

PROFESSOR RUSSELL'S INFINITE.

In *Scientific Method in Philosophy*, p. 159, Prof. Bertrand Russell says, "In the present lecture I wish to state and explain the problem of infinity, to show how it arose, and to show the irrelevance of all the solutions proposed by philosophers."

Again, p. 179, "The difficulties of infinity are of two kinds, of which the first may be called sham, while the others involve, for their solution, a certain amount of new and not altogether easy thinking. The sham difficulties are those suggested by the etymology, and those suggested by the confusion of the mathematical infinite with what philosophers impertinently call the true infinite."

Professor Russell says the infinite is a problem in philosophy. But philosophers have been unequal to their problem. They are "impertinent" when they do not accept the work of mathematicians in the field of philosophy. As I understand Professor Russell, we get the infinite in a series and in a certain kind of number. Failure to accept the infinite as a mathematical series is impertinent.

I have no desire to be "impertinent," but I must admit that Professor Russell's infinite is not impressive. $2/7$ involves a series, 285714, 285714, 285714. This is a series that repeats itself in six terms. That is, the process of dividing two by seven cannot be completed. As far as we go in the dividing, the results are clear, exact, correct, and finite. Manifestly the $2/7$ is not infinite. It is less than one. The dividing is not infinite. It is simple, clear, and correct. The series is a six-term series. It is different from a one-term series. The six terms are not infinite. They are old friends, quite finite. And two or several of them will not be infinite. The mystery lies in the repetition.

If one insists that a fraction of 1 is infinite, I know of no mode of stopping him. But this much is clear. Before we can proceed satisfactorily, we must agree upon our term. What are we to understand by infinite?

Infinite is one of a verbal trinity used to designate reality. Reality is infinite, absolute, universal. These words mark different functions of reality and have been uncovered by separate lines of intelligent activity. As I understand, we owe the knowledge that reality is infinite to mathematics. Mathematics is the "look-out" on the bridge. It reports the infinite in the offing. It tells us nothing

of the crew, the cargo, or the life history of the craft. What is the content of the word infinite? What does it tell us of reality?

Let us consider. Take a stick an inch long. This stick is wood. But wood is not stick. Stick is more than wood. It is wood and two ends. The ends are not wood. And yet the stick cannot be without the ends. What are the ends? The end is the complete absence of wood. A stick is wood and then the wood must cease. Both are essential. And neither is in any sense the other. Ends are not sticks. Wood is not stick. We may say the stick is wood, wood interrupted or broken. But there is no wood at all in the end. Nor can the wood be the cause of the end. In no sense is end wood, or the wood end. Yet both are necessary to the stick. That is, the stick of wood depends upon something that is not wood, that has nothing in common with wood, that lies outside and beyond wood. This is what we understand by finite. Finiteness is dependence. Dependence is that which has its existence, its *Dasein*, outside itself. It is what it is because of its other. And its other lies over against it, beyond it. The end of the stick does not enter into the stick. It is outside. And it differentiates the stick. That is, the stick as stick, as length, depends upon the end. That is, it is what it is because of its outsideness. The difference between a foot-stick and a mile-stick is in the ends. That which is not the thing, the outside, becomes important. Outsideness may be place, distance, or form.

If I offer a sample of distance, can you tell whether it is taken from the inch, the mile, or the ten miles? Outsideness has no quality. There are no marks that differentiate it. Place, distance, form are indifferent to their content. The difference is no part of the space. The space in an inch is exactly the space in a mile. We do not break it or cut it or end it. The object occupying the space is moving along the distance that is stopped. If the moving body is stopped here, it is an inch; there, it is a mile. The space on one side of the moving point is exactly like the space on the other side. We do not stop the space, but the moving body. That is, we have in space a case where the moment of negation is absorbed. Space is not affected by change, by ends, or differences.

The finite we saw to be that which is dependent upon its opposite for its *Dasein*. The infinite is that which is not subject to change, not affected by it at all. In the infinite the difference or change is *aufgehoben*, contained as a moment.

When the moment of negation ceases as difference, we get beyond the finite. We have the infinite. Space uncovers this function of reality. It was the merit of the geometers to get this vision. It was a master vision and has brought wide service to man and mathematics.

The infinite is not any definite space, but space as not affected by interruption. Space does not absorb the interruption. It maintains itself beyond the reach of interruption. That is, as Hegel would say, space negates the moment of negation. The interruptions remain to carry on their business. Hence to set up the infinite as full reality is to practise idolatry. What we wish is an infinite that not only resists change but absorbs all change. An infinite in which change is a moment of itself, ceases to be in any sense different. Here we uncover the basis for the true and the false infinite.

Let us return to our example, $2/7$. This is called an infinite series. The $2/7$ is manifestly not infinite. It is dependent upon one and is less than one. The division is not infinite. It is definite, clear, correct. We get 285714, 285714, and so on as long as our patience and industry sustain us. It is a six-term series. If we take $4/14$, we have the same situation. The mystery is not the $2/7$, nor the division. It is rather that which the division cannot reach. Try as we may, it eludes us. That is, it is not affected by the process. It is beyond the reach of the process. It lures the process of division, and yet is utterly beyond it. It maintains itself permanently beyond the process. To this extent the process is negated. But the process of division is not absorbed. The two stand over against each other.

We reach this conclusion: The $2/7$ are not infinite; the six terms are not infinite; yet the infinite is present compelling the repetition, yet remaining beyond its reach.

The case is this way: Reality is infinite. The humble fraction, $2/7$, shows the infinite. This discovery we owe to mathematics. This is a high service to philosophy. Philosophers are appreciative and publicly give thanks.

Columbus discovered America, and returned to Spain to die. Is it "impertinent" in us to explore his discovery and unfold its riches? We find the infinite in mathematics. That stands. But the infinite of mathematics is a lonely gentleman. He takes no part

ever
like

infi
of
this
Rea
ver
Phi
won
of

is g
text
and
Can
the
beca
mag
part
othe
con
ever

bein
finit
gibl
in a
than
Enc

"Sci

even in the process that uncovers him. Verily "His ways are not like our ways."

It is true that we have a chapter of civilization built upon this infinite of geometry, a static stage marked by permanent separations of classes. But it is equally true that civilization has passed beyond this stage. And so the infinite has a career. There is an evolution. Reality is infinite; reality is also absolute. Further, reality is universal. And so it appears that philosophy has a task of its own. Philosophy is not ungrateful to mathematics. But it regards the work of mathematics as quite preliminary, hardly more than that of Columbus, so far as its own problem is concerned.

H. H. WILLIAMS.

UNIVERSITY OF NORTH CAROLINA.

INFINITY AND THE PART-AND-WHOLE AXIOM.

DEFINITIONS OF THE FUNDAMENTAL ENTITIES OF GEOMETRY.¹

The definition of *infinity* as endlessness, or as a quantity that is greater than any assignable finite quantity, is given in all the text-books on elementary mathematics with which I am familiar, and, so far as I know, was the only one used in this science until Cantor introduced his theory of transfinite numbers. Thus defined, the term is perfectly intelligible and in accord with common sense, because every intelligent person is familiar with the fact that every magnitude is divisible, at least mentally, into an endless number of parts, or can be increased to any other magnitude by adding to it other magnitudes of the same kind, and the human mind cannot conceive how either of these processes could be brought to an end, even if it were continued to all eternity.

Modern mathematicians, however, claim to have reasons for being dissatisfied with this simple definition, and are defining infinity in a manner that not only is not simple and perfectly intelligible, but, so far as I can see, violates Euclid's ninth axiom which, in accord with common sense, proclaims that the whole is greater than its part. Thus, the author of the article on "Number" in the *Encyclopædia Britannica* (11th edition, Vol. XIX, p. 847), defines

¹ The material of this article will be embodied in a chapter of a book on "Science, Truth, Religion, and Ethics" which I am preparing for publication.

an aggregate as *infinite* when it is equivalent to a part of itself, defining equivalence as a one-to-one correspondence between the elements of two aggregates. Equivalence in this case means, therefore, equality of the number of elements in the whole and in the part. In plainer words the new definition is given by Prof. C. J. Keyser (*The New Infinite and the Old Theology*, pp. 49-50) as follows: "The concept of infinity... is this: namely, a... multitude or an aggregate or a collection of elements... is said to be *infinite* if and only if the collection... contains a part, or a subcollection, that is numerically equal to the whole." Here the term equal again applies to the number of terms, or elements, in the whole and in its part. In this definition it is assumed that the number of elements in the part of the infinite aggregate is the same as in the whole of it, so that it is possible to arrange them in a one-to-one correspondence, at the same time assuming that the part contains less elements than the whole, since otherwise it would not be a part only of the latter. The first of these assumptions violates the part-and-whole axiom, while, taken together, the two assumptions contradict each other, and are therefore again contrary to common sense.

The question then arises, What are the reasons for defining infinity in such a, to say the least, remarkable manner? In other words, Are there truths or facts that are inadequately covered by the old definition of infinity? Or, in still other words, Are we really familiar with aggregates whose parts contain as many elements as the aggregates themselves?

Before examining the reasons that are advanced for justifying a new and strange definition, it should be noted that the idea of endlessness of countable terms, or of being greater than any assignable finite quantity, is entirely left out from the new definition, and that no special allusion is made to the implication that the numerical value of each of the elements should remain the same whether they are in the whole or in its part. Evidently the framers of the new definition deemed it unnecessary to emphasize these two requirements, considering them self-evident. But then an aggregate classed as infinite by the new definition is, by virtue of its being boundlessly large, also infinite according to the old definition, while what is finite in the old classification is also finite in the new. Hence even assuming that the new definition is valid, its real value would consist in the supposedly valid claim that, when the number of elements in an aggregate is larger than any assignable finite num-

ber,
of in
claim
comm
critic

endl
are
The
Scie

of r
exan

The
unde
of m
they
any
the
tinu
even
is a
smal
fore
bers

sum
natu
bers
or a
bers
abov
sens
able
num
ing
of t
it is

ber, its part is numerically equal to the whole. The old definition of infinity could therefore be left unchanged, merely advancing the claim that infinity possesses the remarkable ability of violating a common-sense axiom with impunity. We may, however, pass this criticism as mere quibbling about the wording of a definition.

Admitting, then, that every infinite aggregate must contain an endlessly large number of elements, let us examine the reasons that are advanced for the inadequacy of the old definition of infinity. These reasons may be reduced to three (cf. Bertrand Russell, *Scientific Method in Philosophy*, pp. 155-208).

The first reason is the relation between certain infinite series of numbers of which the following is the most commonly cited example:

$$\begin{array}{l} 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots n \\ 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, \dots 2n \end{array}$$

The first series contains the natural numbers in succession, while under each of these numbers is placed its double. The number of numbers is evidently the same in both series no matter how far they are continued; hence when n becomes infinite, i. e., larger than any assignable finite number, the number of numbers must also be the same in both aggregates. But since the top series, when continued to infinity, should contain all natural numbers, both odd and even, while the bottom series contains only even numbers, the latter is a part of the former and the number of terms in it should be smaller. The number of numbers in the bottom row seems therefore to be both equal to and smaller than the number of all numbers.

In this argument there are two fallacies. The first is the assumption that the upper series can be conceived as containing *all* natural numbers. Since the series consists of countable finite numbers, and since every finite number may be increased by adding to it 1 or any other finite number, the phrase "the number of all the numbers of an infinite series" is an absurdity. As was pointed out above, for an aggregate to be infinite, whether in the new or old sense, the number of elements in it must be larger than any assignable finite number. This implies the possibility of increasing the number of terms to any extent and the impossibility of constructing or contemplating *all* of them. The *sum* of an infinite collection of terms may be a finite number, but the *total number* of terms in it is a meaningless phrase: if there were such a total number it

would be finite, and not infinite. The question whether or not there are so-called *transfinite* numbers lying beyond the range of the natural series is not involved in our problem, because, even if they existed, there is no bridge by which one could pass to them by traveling along the series of countable numbers. We are dealing with simple finite numbers, and the properties of numbers which may or may not have existence outside the domain of finite numbers have no bearing on the latter. We must stick to our series of finite numbers, and must not introduce considerations of properties other than those characterizing such numbers.

The second fallacy consists in the assumption that, when continued to infinity, the lower series will contain no other elements than those present in the upper, so that eventually it must become a part of the latter. Since it is not possible to compare individually all the terms of an endless series, the only way one can tell what becomes of our two series as they stretch into infinity is to examine whether or not finite parts of the lower series exhibit a tendency to merge in the upper with the increase in the number of elements of the former. If this be done, it is readily seen that, while both series, when purposely arranged in the above manner, necessarily have the same number of terms no matter how far they are pursued, and while the number of terms common to both constantly grows the more elements we take, the number of elements present in the second but absent from the first series, also becomes greater and greater the further we advance. Thus, among the first six terms of the second series there are the three numbers 8, 10, and 12 that are not to be found among the first six terms of the first. In the group of the first ten terms of the second series there are the five numbers 12, 14, 16, 18, and 20 that are absent from the group of the first ten terms of the first. In general, the greater the number of terms taken, the greater will be the number of elements that are found in the second, but not found in the corresponding part of the first series. Since the presence in the second series of even a single element that is not present also in the first destroys the possibility of the former to be a part of the latter, and since the number of elements that are present in the second series exclusively increases with the number of terms taken, the tendency of this series ever to get drowned in the first becomes smaller and smaller the further we advance. Hence the second series cannot

bec
beg

ing

sho
fir
rel
of
to

sen
rest
tion
of t
uni
seri
fir
it h
seri
by t
posi
to
dire
thei
sec
cont
dista
can

a pa
assu
natu
conf
capa
axio
by L
used
of t
by F

become a part of the first even at an infinite distance from the beginning of the two aggregates.

Another argument. The lower series may be given the following form:

$$2(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots, n),$$

showing that the sum of the second series is double the sum of the first for the same number of terms. Hence at infinity, too, this relation must persist. But if at infinity the second became a part of the first, the sum of the latter, when n becomes infinite, ought to have the greater value.

Still another argument. The two series may be taken to represent the manners of motion of two bodies starting to move from rest, at the same point, at the same time, and in the same direction, the first with a uniform velocity of one unit of space per unit of time, and the second with a uniform velocity of two of the same units of space per same unit of time. The numbers in the top series will then represent both the numbers of units of time the first body has been in motion and the numbers of units of space it has covered during that time, while the numbers in the bottom series will give the numbers of the same units of space covered by the second body during the same time. To assume that at some position along the two series the second merges into the first amounts to assuming that, without changing their relative velocities and directions, both bodies will after a time be at the same distance from their common starting-point. But it is perfectly evident that the second body could never be overtaken by the first, that, on the contrary, the longer the bodies move the greater must become the distance separating them, so that even after an infinite time they cannot be at the same point.

Finally, the assertion that at infinity the second series becomes a part of the first is at best based on an assumption, namely, on the assumption that there is a number expressing the total number of natural numbers, and on the basis of this assumption we come in conflict with a common-sense axiom. As the assumption is incapable of proof, one may reverse the argument, and rely on the axiom for disproving the assumption. This is just what was done by Leibniz who, relying on the inviolability of Euclid's ninth axiom, used the very same two series given above for justifying his denial of the existence of a number of all natural numbers. As quoted by Russell (*loc. cit.*), the following are the words of this mathe-

matician: "The number of all numbers implies a contradiction, which I show thus: To any number there is a corresponding number equal to its double. Therefore, the number of all numbers is not greater than the number of all even numbers, i. e., the whole is not greater than its part." We have then the choice between a common-sense axiom and a mere assumption conflicting with each other. In my humble opinion, the choice is like that between the Rock of Gibraltar and a bed of quicksand as foundation for a castle.

Russell's attempt to answer the argument of Leibniz by replacing "greater" by the phrase "containing a greater number of terms" is futile, because the number of all numbers indicates the total number of terms in the natural series, and, since the latter consists of both odd and even numbers, the number of terms in it necessarily is greater than the number of even numbers in it. In fact, the whole difficulty of the framers of the new definition is due to confusing the number of even numbers in the natural series itself with the number of even numbers purposely placed as a new series that is in a one-to-one correspondence with the numbers of the natural series. In the first case the even numbers really form part of the natural series, but their number obviously is only one half of the total number of terms in the series, no matter how far the latter is continued, while in the second case the even numbers form an independent series which, like any other infinite series of numbers of things, may be arbitrarily arranged in a one-to-one correspondence with the natural series. In other words, if in forming the lower series we would use only the even numbers found in a given portion of the natural series, we would always have only one half enough terms to form a one-to-one correspondence with all the terms of that portion no matter how large the latter is; in this case the number of terms in the part would be smaller than in the whole, as it should be. On the other hand, in order to get such a number of even numbers as would equal the number of terms in that portion, we would have to go outside the latter, so that the second series containing elements foreign to that portion would not be a part of it. The assertion that in the natural or in any other continuous endless series of numbers the number of elements is the same in the whole and in its part is therefore, so far as I can see, erroneous.

The second reason for considering the old definition of infinity inadequate and for ascribing to infinite numbers a property that

viola
of p
the f
a ra
dial
radi
the c
succ
feren
of th
the n
But
form
poin
circu
pair
into
hold

poin
to th
is sh
stra
Ever
thou

of th
we a
circl
circu
a si
of p
of t
the
at th
and
an in
para
coim

violates the part-and-whole axiom, is the identity of the number of points in all lines, regardless of their lengths. This I show in the following manner: Draw two concentric circles, and assume that a radius common to both is uniformly moving, like a hand on the dial of a clock, along the two circumferences. Every time the radius touches a point on one circumference it touches another on the other, so that when the circuits are completed and the radius has successively come in contact with all the points of both circumferences, it has touched exactly the same number of points on both of them, and no point has been touched more than once. Hence the number of points in the two circumferences is exactly the same. But since the inner circumference is shorter than the outer, the former represents only a part of the latter, and the number of points in it should be smaller. But it was just proved that both circumferences contain the same number of points. Since every pair of lines, straight, curved, or broken, can be mentally converted into two concentric circles, the equality of the number of points holds good for all lines without exception.

Usually the same proposition is proved as follows: From a point on one of the sides of a plane triangle draw a line parallel to the base. The part of this parallel within the triangle obviously is shorter than the base. Now assume that an infinite number of straight lines are drawn from the vertex to points on the base. Every one of these lines will intersect the parallel, so that the latter, though shorter than the base, must have the same number of points.

That in this argument is hidden a fallacy somewhat akin to that of the famous paradoxes of Zeno can be readily shown. Suppose we assume inside the smaller circle an infinite number of concentric circles, one smaller than the other. The lengths of the concentric circumferences will continually diminish, the last one changing to a single point coinciding with the center. But since the number of points is the same in all the circumferences, and since the lengths of the latter decrease *continuously*, it ought also to be the same in the final circumference, which is a single point. We thus arrive at the absurdity that the number of points in all lines is both infinite and equal to 1. Similarly, we may assume within the above triangle an infinite number of lines parallel to the base. The lengths of these parallels within the triangle decrease *continuously*, the last one coinciding with the vertex, which is a single point.

In order to answer this argument it is necessary to answer three

distinct questions: (1) Why is the number of points in all lines infinite? (2) Why is the value of this number the same for all lines? (3) How can this number be the same for a line and for a center or a vertex?

The first question is answered by considering the fact that lines have length, while geometrical points are by their very definition dimensionless. Representing the lengths of different lines by positive numbers and the longitudinal dimension of a geometrical point by zero, the number of points in any line is given by the quotient obtained by dividing zero into a positive number, which quotient is always infinite. This shows that the point-and-line argument is entirely irrelevant to the problem of the validity of the part-and-whole axiom. The infinity of the number of points in all lines is not due to a peculiarity of certain infinite or transfinite magnitudes, but merely to the fact that lines occupy longitudinal spaces, while points do not. The only difference between lines that is involved in the case is a difference in length, a spatial difference, but since mathematical points need no spatial houses to reside in, their number in a given line necessarily is as independent of the length of the latter as the number of thoughts or feelings one may have is independent of the size of one's head or the width of one's chest. If drops of water occupied no space, an infinite number of them could find room and to spare in a thimble. Does that violate the part-and-whole axiom?

The answer to the second question is that the whole difficulty is due to an inadequate definition of the geometrical, or mathematical, point. Euclid's time-honored definition of a point is: *A point is what has position but no magnitude.* The definition contains two flaws. Its first flaw is that it fits many entities having nothing whatever to do with mathematical points, such, for instance, as the feelings of living beings and the numerous properties of things in general. Thus, one may say that whiteness and sweetness have position in white and sweet things but no spatial magnitude, since a grain of sugar is as white as a ton of it and several hundred times less sweet than a grain of saccharin. Similarly, love or hatred has position in the lover or hater but no spatial magnitude of its own, since a midget may have a greater love or hatred than a brooding-nagian.

The second flaw of the Euclidean definition of a point consists in that it fails to locate a single geometrical point in a given

part
ceiv
of s
lost
but
ings
ther
but
a pl
for
rela
pyin
This
noth
wor
ical
shal
with

is to
spac
to h
whe
geon
or c
assign
is m
line
men
orig
with
of p
into
men
i. e.
tion
of
divi
leng
draw

part of space or on a given line. Since spaceless places are inconceivable, there is no room for a single mathematical point outside of space, while the moment it is provided with the latter it gets lost in it, the smallest conceivable part of space containing not one but an infinite number of mathematical points. In the case of feelings or properties their location is in the beings or things possessing them. A geometrical point, on the other hand, is not a property of, but a location in, space, and Euclid's definition does not provide a place for it. To be sure, there are entities needing no space at all for existence. Thus, the abstract ratio of two numbers and the relation of parentage between father and son exist without occupying any space, either of their own or of other things or beings. This, however, is due solely to the fact that such entities have nothing in common with space; they could exist even in a spaceless world if such a world were capable of existing. But a mathematical point is either situated in space or is inconceivable. How, then, shall we house a single geometrical point in space which, in accord with common sense, is compact, i. e., devoid of interstices?

So far as I can see, the only way to overcome the difficulty is to define a point as follows: *A point is an infinitesimal part of space which, for the sake of mathematical argument, is assumed to have shrunk to nothing, to a spatial zero, leaving a position just where the given part of space is assumed to have disappeared.* A geometrical point is therefore the limit which a definite part of space or of a line approaches by gradually becoming smaller than any assignable extension. As long as the point has spatial existence, it is mentally divisible into the same infinite number of real parts as any line, surface, or solid, but when by agreement, for the sake of argument, we let it die as a spatial entity, the part of space from which it originates and all the parts into which it is mentally divisible expire with it. The assertion that all lines have the same infinite number of points merely means that they are all divisible, at least mentally, into the same infinite number of real parts, each of which may be mentally, i. e., by agreement, be converted into a mathematical point, i. e., a position without magnitude. As a matter of fact, all illustrations going to show the identity of the number of points in lines of different lengths prove nothing more than that the lines are divisible into the same number of real parts of equal or unequal length. Thus in the above triangle the infinite number of lines drawn from the vertex to the base merely divide the latter and all

the parallels to it within the triangle into the same infinite number of real parts, and not into geometrical points, because no line, no matter how short, can be reduced to a mathematical point by actual, even everlastingly continuous, division. Similarly, the radius moving along the above circumferences marks not mathematical points, but degrees, minutes, seconds, and real fractions of the latter, all of which are real lengths. The number of these is the same in all circumferences, but their lengths vary with the lengths of their radii. A radius progressing by mathematical points only would not move at all, because no number of such points, be there ever so many of them, would by addition give the smallest imaginable fraction of a degree.

The answer to the third question is now obvious. As long as the parallels, the circumference, the vertex, and the center have spatial existence, they all contain the same endless number of real, at least mentally, measurable parts, but when for the sake of mathematical argument we kill the center and the vertex we kill with one stroke all their parts and all the lines coinciding with them. There is nothing in this description that makes a part of an aggregate contain the same number of the same elements as the whole, because, while the number of the latter is the same, their values in the part and in the whole are different. And there is nothing in the above definition of a geometrical point that would invalidate any proposition in geometry, since, in accord with this definition, the mathematical point is just what the mathematician wants it to be—a position without magnitude. The derivation of the geometrical point from real space would not impart to it greater spatial reality; it merely provides it with a location.

It is hardly necessary to point out that, if the above definition of a geometrical point were adopted, a line could be defined as a part of space of such infinitesimal width and thickness that, for the sake of mathematical argument, it is considered to be unidimensional; and a superficies as a part of space one of whose dimensions is so infinitesimally small that, for the sake of argument, it is considered to have only two dimensions.

The third reason for finding fault with the definition of infinity as merely boundless greatness consists in the supposed difficulties involved in the famous paradoxes of Zeno. Notwithstanding the fact that the fallacies of Zeno's arguments have been pointed out long ago, some philosopher-mathematicians still seem to be per-

plex
Zen
and
esse
will
Ach
dox

the
will
and
but

only
ity.
spa
adm
van
com
pac
atte
of
pac
and
from
day
of
of
tryi
pro
be
of
of
sar
the
and
pos
a m
adv

plexed by these paradoxes. I shall therefore put the answer to Zeno's argumentation in a form which is, in my opinion, conclusive, and which, so far as I know, has not been used before. Since the essence of the difficulties is the same in all of these paradoxes, it will suffice to treat one of them. We shall pick out the problem of Achilles and the tortoise. As quoted by Russell (*loc. cit.*, the paradox is stated as follows:

"Achilles will never overtake the tortoise. He must first reach the place from which the tortoise started. By that time the tortoise will have got some way ahead. Achilles must then make up that, and again the tortoise will be ahead. He is always coming nearer, but he never makes up to it."

The answer is as follows: Achilles is *admittedly* capable not only of motion in a general way, but of motion with uniform velocity. In other words, he is admittedly capable of covering the same space within the same time. In pursuing the tortoise he is therefore admittedly capable of progressing by *whole* paces. He may advance by fractions of a pace if he likes or needs to, but he is not compelled to do so. Hence in going after the tortoise he will make pace after pace until he is on the top of the tortoise. If Achilles attempted to perform the impossible task of counting the fractions of a pace, i. e., advancing first by one pace, then by one half of a pace, then by one fourth of a pace, then by one eighth of a pace, and so on, he would get no further than a trifle less than two paces from his starting-point, even if he kept on moving to the end of his days. Assuming the tortoise to be no further from Achilles than two of his paces, it could quietly lie down and, without the slightest fear of ever being captured by the Greek hero, "smilingly" watch him trying to verify the value of the sum of an infinite geometrical progression whose first term is 1 and whose ratio is $1/2$. This may be illustrated as follows. Suppose you wish to measure the length of your room. You would get a yardstick and lay off the *whole* of it consecutively along the floor, finishing the operation, if necessary, with a fraction of the stick. But suppose you first laid down the whole of the stick, then one half of it, then one fourth of it, and so on, you would never cover the whole length of the floor, supposing its length to be no less than two yards, even if you lived to be a million times as old as Methuselah. Since Achilles is as free to advance by whole paces as you are free to advance by whole

yardsticks, he is bound to catch the tortoise. The problem of infinity is therefore not involved here at all.

The résumé of this article is, that the essential meaning of *infinity* is endlessness, or being larger than any assignable finite magnitude, that this should therefore be the definition of the term, that whatever other properties infinite aggregates may possess should be given as additional characteristics, provided it can be proved that these aggregates really possess such characteristics, and that no aggregate is at present known which contains as many elements of the same value in its part as in the whole of it. There can, of course, be no objection to *assuming* the existence of *transfinite* numbers doing violence to our good old axiom, provided they are treated like the imaginary quantities of algebra, and provided such an assumption possess real merit in the theory of numbers. Whether or not this is the case, I leave it to greater specialists in mathematics than I am to decide.

H. M. GORDIN.

NORTHWESTERN UNIVERSITY.

CURRENT PERIODICALS.

In *Scientia* for April, 1919, W. E. Harper, of the Dominion Observatory, Ottawa, sketches the part played by the spectroscope in widening our astronomical horizon. He dwells upon the method, the new determination of the motion of the solar system through space, the discovery of spectroscopic binaries and their orbits, and the problems of luminosity. "The structure of the universe is the problem toward the solution of which many lines of research are contributing," and "probably from none of them may we expect more than from the spectroscopic, because the data regarding the physical constitution and the motions of the heavenly bodies are secured at once."—Evolution and sexuality is the subject of E. Rabaud's contribution to this number. He deprecates the predominance of the attention paid to the morphological aspect of questions which are much more appropriately concerned with the physico-chemical differentiations which cause the morphological change—differentiations which at times are almost ignored. After discussing the work that has been done up to the present on the genesis and nature of sexuality he proceeds to dwell upon what is known of its

consequences, and how far the latest work throws light upon such vexed questions as sex-determination, and the importance in the evolution of organisms of a sexuality which may be not merely useless but even harmful to the well-being and continuance of a species.—E. Lattes takes up once again the story of the Etruscan enigma, of that obscure and much-discussed tongue which has been the subject of an age-long controversy among philologists. He is of the school which holds that there is sufficient material in hand to enable a sober judgment to be formed as to these remnants of Etruscan speech, and he holds the Italian gates against all assault.—Libby's *Introduction to the History of Science* gives to Aldo Mieli an opportunity of pointing out the pitfalls that beset the feet of those who attempt to give in a short compass an accurate and synthetic account of the development of knowledge and of man's conquest of nature.

In *Scientia* for May, 1919, Gino Loria begins the first of two articles on the past and present of mathematics in Spain.—Harold Thomson in his paper on "The Planet Mars" takes the opportunity of reminding all concerned that the astronomer has no special function to pronounce on the habitability of other worlds. He pays a well-deserved tribute to the systematic work of regular observers under the British Astronomical Association. He suggests that the destruction of great forest areas, the change in aspect of vast tracts from the green of crops to the red of ploughed-up soils, the contrast between the golden corn and the fallow fields, are among many of the causes which would account for markings hitherto attributed to the existence of artificial "channels." But it is only by the multiplication, digestion, analysis, and coordination of observations that we can hope to obtain a definite answer to the problem of the habitability of the planet.—G. Bohn's paper on a new orientation in biology renews the attack on the doctrines of "finalism" with which his name is associated. After sketching the general trend of biological speculation as dominated by the tendency to specialize—"We have anatomists, physiologists, and the like, but no biologists"—and dwelling on the reception accorded in Germany, "the land of illusions," to the theory of "substance-properties," he comes to the secular controversy on the hereditary transmission of characters, and points out that the primordial problem must be the determinism of characters. "A living being is not merely an aggregate of chemical substances, it is also a system of movements, and therefore a system of forces." This view of living beings and their external

movements as the expression of the play of internal forces becomes in his hands a fruitful hypothesis, and leads to the substitution of "internal formative movements" for the "formative substances" of Sachs. He closes with the prophecy: Physics and chemistry were revolutionized in the last century by the conception of molecular motion; the present century will see a revolution in biology, and the external forms and movements of living beings will be regarded as the resultant of internal forces, the manifestations of the energy of living matter, and these will have become accessible to scientific analysis.

In *Scientia* for June, 1919, Gino Loria concludes his sketch of the past and present of mathematics in Spain, dwelling in the main upon the work of Echegaray, Torroja, and Garcia de Galdeano. and the revival which, under the notably successful leadership of J. Rey Pastor, has already achieved such promising results.—That radiation, necessarily present in material systems (in virtue of their temperature) is to be taken into account in modern chemical science, is the subject of a valuable paper by W. McC. Lewis, entitled "Radiation, the Fundamental Factor in All Chemical Change." He shows how from two different standpoints the same general conclusion has been reached by the work of Arrhenius on the one hand and Marcelin and Rice on the other—that the velocity constant must be an exponential function of the temperature. An increase in the internal energy of a molecule accounts for chemical velocity and the influence of temperature on the velocity constant. Whence is derived the additional increment of energy? The source is radiation.—Philology is represented in this number by A. Meillet's paper on grammatical style and the elimination of flexion. He discusses the influence exerted upon the development of a language by the mentality of those who speak it, their degree of civilization, and by certain checks to that action arising from causes which are inherent in the system of the language in question, and which may affect it in quite unexpected directions.

* * *

In *Science Progress* for April, 1919, we have fifty pages giving the usual summaries with ample references of "Recent Advances in Science."—"Recent Work on the Spectra of X-Rays," by W. H. Bragg, is an article of exceptional value and interest, in which the author remarks that now the war is over "we are all in the position of learners again, and it may be convenient to restate the first prin-

ciples of the subject." Tribute is paid to the value of recent work in America on the improvement of the spectrometer used in analyzing the rays, and investigations on the law governing the conversion of X-ray energy into electron energy are analyzed and discussed. Such laws "are sure to throw light both on the processes of radiation and on the nature of the atoms from which radiation proceeds."—The effect of X-rays on living matter is the subject of a paper by S. Russ on "Growth and Division of Cells as Affected by Radiation." The effect of variation of the quantities of radiation is described. An "intrusion" of mathematics leads to the conclusion that if irradiation of a tumor results in diminution in cell-growth, the processes that govern the growth of the young cell to the mature form have been somehow or other subjected to change. What changes are set up in the metabolism of the cells after irradiation have yet to be discovered with certainty.—Ingvar Jørgensen and Walter Stiles deal with "The Scientific Aspects of Cold Storage," pointing out the direct bearing of the principles evolved in pure science on a subject of momentous economic importance. To this, among many other problems of the industrial order, it is now possible to assign a definite research basis.—In "The Ice-Age Question Solved," R. A. Marriott disinters the astronomical work of Drayson on the precession of the equinoxes. He aims at freeing "geologists from their astronomical trammels by convincing them that the dogma which has barred the way to an understanding of the glacial period is now practically removed." He begins by rehearsing the geological facts agreeing with Drayson, and among other claims announces that Drayson's one movement of the annual motion of the pole will account for the progressive decrease in obliquity and its rate, the time and duration of the glaciation, the changes in co-latitude of the stars, and the error in sidereal time. We may foresee in the near future a fine display of wigs on the green.—The defects from the mathematical point of view of the last edition of the *Encyclopædia Britannica* form the subject of an interesting and extremely suggestive essay by Philip E. B. Jourdain. It might be worth while, were compilers of encyclopedias likely to take any notice of it, to make a list of the biographies in which the eleventh and other editions were deficient. That such a list is necessary is obvious enough if we reflect that not till the last edition was there a life of Cauchy included, and not even then is there a life of, to mention but one name on such a list, George Green. γ

BOOK REVIEWS AND NOTES.

LOGIC AS THE SCIENCE OF PURE CONCEPT. By *Benedetto Croce*. Translated from the Italian by *Douglas Ainslie, B.A. (Oxon.), M.R.A.S.* London: Macmillan and Co., Ltd., 1917. Pp. xxxiv, 606. Price 14s. net.

This volume is the second of a series of four by Croce on the philosophy of the spirit—the other three being: *Esthetic as the Science of Expression and General Linguistic*; *Philosophy of the Practical: Economic and Ethic*; and *The Theory and History of History*. The last has not yet been published; the other three, including the volume under present consideration, have been translated into English by Mr. Ainslie.

The book is divided into four parts: I. The Pure Concept, the Individual Judgment, and the *a priori* Logical Synthesis; II. Philosophy, History, and the Natural and Mathematical Sciences; III. Error; IV. Historical Retrospect. The first two parts are more important than the last two, considered as expositions of the author's general attitude.

According to Croce, logic is the science of pure concept—pure as distinguished from the fictitious concept, or pseudo-concept. The pure concept is characterized by three properties: (a) universality and transcendence, (b) concreteness—i. e., applicability to the actual world, (c) expression—which is the quality of being enunciated in some form of language, not vocal necessarily. Examples of pure concepts are beauty, quality, final cause. Pseudo-concepts are such as lack one or more of these properties; more particularly, they are either abstract, that is, lack instances (e. g., the concept *triangle*), or they are mere class-names, lacking transcendence and universality (e. g., the concept *house*, which, being directly representative of its object, lacks a special content of its own and is of limited application). Pure concepts include their opposites—thus "beauty is such because it has within it ugliness" (p. 97); "if God had not Satan in Himself, He would be like food without salt" (p. 98). But when taken thus complete, the concepts are distinct from one another. The expression of the pure concept constitutes the judgment of definition; definition, in this instance, is true absolutely, becoming arbitrary when the content of the judgment is a pseudo-concept, as is the case with mathematical definitions.

Parallel with the judgment of definition we have the individual judgment, like "Peter is good," where the subject is a representation and the predicate a pure concept. An individual judgment implies the existence of its subject, existence being one of the attributes of the latter. Corresponding to pseudo-

concepts, we have pseudo-judgments, namely, judgments of classification and of enumeration.

Now, in relation to these distinctions of concept and judgment, distinctions of knowledge appear. There is thus philosophy, which is the thought of the pure concept; there is history, i. e., the knowledge expressed through individual judgments. These two are the only legitimate forms of theory; in fact, they are ultimately identical—the individual judgment is conceptual as well as perceptual in that it contains a predicate, and on the other hand, the judgment of definition is historical as well as universal, for it always is an answer to a question individually conditioned, and the philosopher, in the standpoint that he assumes, is influenced by his temperament and the historical conditions of his time. The argument does not seem very cogent at this point; the historical conditions just mentioned concern the philosopher himself and in no way are constituents of the judgment of definition as such. There are two other forms of knowledge, natural science, depending upon group-names and using the method of classification, and mathematics, which receives expression in abstract judgments. Both of these are illegitimate forms of knowledge, for they depend on pseudo-concepts, in fact they are not forms of knowledge at all, strictly speaking, but really forms of action.

In further chapters the author takes up the topic of error, its nature and its varieties. His view of religion is of special interest; religion is according to him that form of error which consists in the "affirmation of the universal as mere representation," i. e., in the expression of a general character like "universe" through something particular, such as "personal God"; religion is false philosophy, and philosophy is the true religion. The author is particularly severe on what he regards as the error of formal and symbolic logic; he accuses them both of basing their analysis of the concept on its verbal form, pretending to "obtain thought in words." Croce almost betrays a bias against mathematics as when he says of it: "It is a devastation, a mutilation, a scourge, penetrating into the actual world, in which it has no part" (p. 370).

Croce exhibits historical affinities with Kant, whose doctrine of the logical *a priori* synthesis he regards as a philosophical discovery of the highest importance, with Hegel, particularly as concerns the unity of opposites in a concept, and with Vico, whom he admires for the importance which he (Vico) attached to the historical point of view. In his doctrine of the concreteness of the universal, our author comes very close to Bosanquet and others of the same school. The style of the work is very vivid and flowing but it suffers in its excessive dependence on metaphors and the symbolic modes of expression, in general. Speaking of the value of philosophy, he says some very good things; thus (p. 499), "Philosophy brings consolation in its own kingdom, putting error to flight....; but man is not thought alone....philosophy has no handkerchief to dry all the tears that man sheds." Croce denies that philosophy and logic are only for the expert and that they are absolutely distinct from the field of common sense: "As the hero is not outside humanity, but is he in whom the soul of the people is concentrated and made powerful, so poetry, philosophy, science, and history, aristocratically circumscribed, are the most conspicuous manifestations attained by the elementary forms of acquaintance themselves" (p. 253). And, in a vigorous statement of the view that the philosophic thinker never attains finality, he says: "To any philosopher,

as to any poet, the only works of his own that bring true satisfaction are those that he has still to do" (p. 317).

Perhaps the most striking feature of the book is the relegation of natural science and mathematics from the field of theory to that of action. This attitude is based upon the view that the concepts of science and mathematics are fictitious. But to come to a specific point, it is not wholly clear why a concept like "house" should not be regarded as possessing universality; having a definite meaning, like all other concepts, it applies to a definite set of objects, but the number of these is not limited, for the concept house applies to all possible, as well as actual, houses. And in general, instead of adopting a definition of science and mathematics which is also an evaluation of them, would it not have been more desirable to distinguish them from philosophy and history by reference not to the mode of knowledge but to the subject-matter—to define, say, history, as dealing with the particular and changing elements and science with the constant and universal elements of reality? Such a definition would not prejudge the question of the cognitive value of that which it defines and indeed would allow full scope for the claims which, undeniably, both science and mathematics make upon the intellect.

The point of view in the author's treatment of logic is thoroughly philosophical and systematic. Assertions in this volume are defended by references to theses expounded and adopted in the author's other works, and theses that are purely metaphysical in purport. The argument moves with that ease—and we may add, dogmatism, on some occasions—which come from the consciousness that any given stage in the argument is completely bound up with and determined by the stages already covered.

In most forms of knowledge, there have been two stages, the philosophical and the scientific, the latter following upon the heels of the former; thus, physics, psychology, chemistry, have all been "philosophies" before, but are sciences at the present. The same movement has appeared in the growth of logical thinking. Hegel is the preeminent representative of the purely philosophical approach to logic, whereas the present tendency to identify logic with mathematics represents the gradual development of logic into an exact science. Croce is a self-conscious upholder of the philosophical approach to logic, and the present work constitutes a forceful, clear, and on the whole, notable contribution to the treatment of logic as a philosophical study. Δ

A DEFENCE OF IDEALISM: SOME QUESTIONS AND CONCLUSIONS. By *May Sinclair*. London: Macmillan and Co., Ltd.; New York: The Macmillan Company, 1917. Price, 12s. net.

This entertaining volume of essays opens with a chapter on the Panpsychism of Samuel Butler. Miss Sinclair traces out Butler's destruction of the theory of "personal identity," and discusses his view that habit and instinct are but memory and that both presuppose knowledge and volition on the part of the individual that displays them. Butler's theories and those of the new psycho-analytic school stand in an interesting relation. Butler, of course, held that the only sane and perfect life is the unconscious life. The psychoanalysts, on the other hand, seem to assume that we must live consciously if we would live well. An interesting account is given of the conceptions made use of by the psychoanalysts and it is shown that Butler's work helps to an

understanding of some of the features of the investigations now being undertaken by this school of psychologists.

Miss Sinclair is very happy in her treatment of mysticism, the chapter entitled "The New Mysticism" being quite a graceful study of the subject. Especially interesting is her discussion of the relation between mysticism and monism. Much trouble is taken to show that it is not the case that "Monism is the offshoot of mysticism, a disease of thought reverting to a savage ancestry" (p. 277) but rather that mysticism owes much more to philosophy than philosophy could ever owe to it. But another aspect of mysticism is also treated, and it is this aspect that is responsible for the increase of interest in the subject which is to be found at the present time. The results of psychoanalytic investigations are applied to the experiences of such mystics as Lady Julian of Norwich, St. Catherine of Genoa, St. John of the Cross, and St. Teresa. We cannot but feel that the interpretations of their religious experiences offered must make them turn in their graves. But it is intensely interesting subject, and the treatment of certain religious experiences in the light of modern knowledge of abnormal psychology seems to be a step in the direction of a healthier point of view with regard to mysticism and the various problems involved.

But the reader to whom the subject of mysticism does not specially appeal may turn to other topics. There is a chapter on vitalism, a chapter on psychology, one on pragmatism and humanism, and one on the new realism. The discussion of the new realism is light and amusing: it is a pity that the treatment of some of the most important points is not as full as the eager reader might desire. Reference is made to the work of Mr. Russell, Professor Whitehead, Professor Alexander, and the American Six Realists, who are amusingly compared to a colony of the Young Men's Christian Association. But why is Mr. Russell's treatise *The Principles of Mathematics* referred to continually as *Principia Mathematica*, which is the name of Mr. Russell and Professor Whitehead's logic? Emphasis is rightly laid on the importance of the method employed in the new realism. Once again it is explained that in the idealistic philosophy, formal logic could not cope with mathematical deductions, and so idealists condemned mathematics. It is indeed stated that "all idealisms, constructive or destructive, are based on the ultimate inability of mathematics to defend its own position" (p. 179). In the course of her defense of idealism our author has therefore to show that, even now, no satisfactory account can be given of infinity and continuity and the various conceptions with which mathematics deals. And in this endeavor she does not succeed. First an account—from the point of view of an idealist, be it remembered—is given of the work of the mathematical logicians on continuity and infinity. In an easy style, our author suggests the kind of way in which Cantor, Russell, and Whitehead maintain that space and time are no longer internally refutable: but one feels that, in spite of brave attempts, it may still be a mystery to some readers how exactly the arguments run. To appreciate the cogency of such arguments, arguments which do in fact establish the thesis that space and time are not internally refutable, some slight knowledge of the modern theory of series is needed, and this is not given in the discussion: the result is that the man in the street is puzzled and the logician is unsatisfied. As a whetting of the appetite for further information, this light-handed treatment of difficult

subjects may be useful. It is amusing to see the extraordinary and paradoxical conclusions at which philosophers can arrive when they try to treat mathematical subjects—which have of course a method of their own, adequate to deal with them—with the methods of the idealistic school. There must be some innate perversity in the minds of nearly all human beings that renders them unwilling to look at the problem of the infinite with moderate carefulness and good-will. Otherwise it would not have been left to the nineteenth century to produce the theory of infinity. And it would almost seem that it was not lack of knowledge of logic but something more fundamental which was responsible for the delay. For, even now, when the subject is ready to give up the secrets of the infinite to any one who takes the trouble to look into it, we find quick and capable thinkers arriving at the most nerve-ruining results about it. Thus our author arrives at five “fine, flourishing contradictions” (p. 256):

1. the contradiction of the infinite regress;
2. the contradiction of the non-serial series;
3. the contradiction of the positionless position;
4. the contradiction of the non-spatial spaces and non-temporal times;
5. the contradiction of the finite infinite.

Who would attempt to boil potatoes with the help of a knowledge of Greek, or to attack the problems of sociology with the apparatus of chemistry? Why then treat the technical developments of modern mathematical logic with the all-embracing intuitions of the idealistic philosopher?

Other aspects of the new realism are considered, especially the question of the externality of relations. Many a subtle and difficult point is lightened (if some are obscured) by an amusing jest or a pretty phrase. An exposition of what is called “The Monistic Theory of Reality” is given and Mr. Russell’s criticism of the monistic theory of truth is examined and pronounced unsound, but as it seems to the reviewer for insufficient reasons. But in spite of her treatment of the new realism, Miss Sinclair has produced an amusing and in parts an interesting book—a book any philosopher would like to read on a Sunday afternoon.

DOROTHY WRINCH.

SLAV ACHIEVEMENT IN ADVANCED SCIENCE. By *Branislav Petronievcics*. Pp.

32. London: The American Book Supply Co., Ltd., 1917. Price, 1s. net.

After the Renaissance the order in time in which nations took part in continuing the work of the ancient Greeks is: Italy, France, England, Germany, Russia. Dr. Petronievcics shows in this pamphlet that “not only Russia can boast of men of science comparable to the greatest among the peoples of Western Europe, but some are to be found even among the other Slav nations” (p. 5). The four men dealt with at some length are the Pole Niklas Kopernigk (Nicolas Copernicus, 1473-1543), the Russians Nikolay Ivanovitch Lobatchevski (1793-1856) and Dimitrije Ivanovitch Mendeljew (1834-1907), and the Serbo-Croat Rudjer Josif Boshcovic, usually known as Roger Boscovitch (1711-1787). The transcription of names is the author’s. It is interesting that (p. 9) Copernicus, in his theory of the earth’s daily motion about its axis, was, “as he himself acknowledged, following in the footsteps of the ancient Pythagoreans. His second principal theory, however, that the earth’s movement around the sun together with all the other planets, was almost

entirely evolved by himself." In the substitution of what Bailly called "a certain probability of the mind felt by a small number of philosophers" for that of the senses (p. 9), he retained many of the ideas of the ancients. The account of the system published by Boshcovic from 1745 to 1758 given on pp. 13-15 is exceedingly interesting, especially in connection with modern work on the exact formulation of the logical principles of physics, and comparatively unknown. Also the accounts of the work of Lobatchevski on non-Euclidean geometry (pp. 19-22) and of the periodic law of chemical elements (pp. 25-30) are well and clearly written, especially the latter. Of course there are many other Slav men and women of science who are perhaps of not quite such great importance (p. 32). There are a few critical remarks which may be made. In the first place, the distinction that the author draws between scientific value and philosophical value (pp. 6, 16, 23, 31) does not seem quite valid. There is really no special domain of inference open to philosophers but closed to men of science and a proposition cannot strictly be false in science and true in philosophy. In the second place, Leibniz's law of continuity does not seem correctly stated on p. 13. There is a misprint for Beltrami's name on p. 18. ϕ

BEETHOVEN. By *Romain Rolland*. Translated by *B. Constance Hull*, with a brief analysis of the Sonatas, the Symphonies, and the Quartets by *A. E. Eaglefield Hull*. London: Kegan Paul, Trench, Trübner and Co., Ltd., 1917. Pp. xx, 244. Price, 2s. 6d. net.

This well-written and well-translated book consists of Rolland's short account of the life and extracts from some of the letters of Beethoven. To this is added an analysis of the musical works by the editor. The book is completed by a bibliography, a classification of piano sonatas, and a complete list of works. There are four excellent illustrations. ϕ

A MANUAL OF MODERN SCHOLASTIC PHILOSOPHY. By Cardinal *Mercier* and Professors of the Higher Institute of Philosophy, Louvain. Authorized translation (eighth edition) by *T. L. Parker* and *S. A. Parker*, with a preface by *P. Coffey*; Vol. II: Natural Theology (Theodicy), Logic, Ethics, History of Philosophy. London: Kegan Paul, Trench, Trübner and Co., 1917. Pp. xvi, 551. Price 10s. 6d. net.

This is a text-book for colleges, written from the Thomist-scholastic point of view in philosophy. The first section of the book—on Natural Theology—is written by Cardinal Mercier and deals with the topic of God: His existence, nature, activities, and work. In going through the arguments put forward, the reader cannot help feeling that the recent discoveries in mathematical logic on the definition of infinity make necessary a reconsideration of the proofs for God, in so far as these assume that a series—whether of causes or of perfections—must have a first or a last term. The section on Logic is also written by Cardinal Mercier, its subject-matter is stated to be the intellectual processes, its purpose, the discovery of the conditions under which these processes lead to the apprehension of truth. The most interesting of all is the section on Ethics (written by A. Arendt and J. Halleux). Here socialism, the institution of the family, the nature of the State, and other similarly urgent problems are discussed in a stimulating fashion. In the section on the History

of Philosophy (by M. de Wulf), prominence is given to the medieval period; notice is also taken of non-Western schools of philosophy, like those of India and China, and there is a brief treatment of contemporary philosophy, with a discussion of Nietzsche, Bergson, and others. The book closes with a synopsis of the theses presented in both volumes of the manual, a glossary of philosophical terms, an index, and an appendix. A

BOOKS RECEIVED.

- Losky, N. O. *The Intuitive Basis of Knowledge: an epistemological enquiry.* Authorized translation by N. A. Duddington, M.A., with a preface by Professor G. Dawes Hicks. London: Macmillan and Co., Ltd., 1919. Pp. xx, 420. Price, 16s. net.
- Law, A. *In Darkest Christendom, and a Way Out of the Darkness.* London: G. Allen and Unwin, Ltd., 1919. Pp. 256. Price 7s. 6d. net.
- Mary W. Calkins. *The Good Man and the Good. An Introduction to Ethics.* New York: The Macmillan Co., 1918. Pp. xx, 219. Price, \$1.30.
- George A. Barton. *The Religions of the World.* Chicago: The University of Chicago Press. [1917.] Pp. x, 349. Price, \$1.50.
- Ludwig Stein. *Philosophical Currents of the Present Day.* Translated by Shishirkumar Maitra, M.A. Vol. II. Calcutta: The Calcutta University Press, 1919. Pp. 237-393.
- William E. Hocking. *Human Nature and Its Remaking.* New Haven: The Yale University Press, 1918. Pp. xxvi, 434. Price, \$3.00.
- George H. Lepper. *From Nebula to Nebula.* (Fourth edition, revised and enlarged.) Privately printed, 1919. Pp. 401.
- Henry Rutgers Marshall. *Mind and Conduct.* New York, Charles Scribner's Sons, 1919. Pp. viii, 236. Price, \$1.75.

OBITUARY.

It is with deep regret that we chronicle the death of W. Max Müller, who lost his life at Wildwood, N. J., on July 12, when taking his daily swim in the ocean. Dr. Müller was born at Gleissenberg, Germany, May 15, 1862. After studying in the universities of Leipsic, Berlin, and Munich, and taking his doctor's degree in the University of Leipsic, he came to the United States in 1888. He soon became known as one of the foremost Orientalists in the country. Among his contributions to various periodical publications one of the most recent was his paper on "Noah and His Family" which appeared in *The Monist* for April, 1919, under the pen-name M. Milman.

As we are going to press the sad news reaches us of the death of Mr. Philip E. B. Jourdain, our associate editor in London. Mr. Jourdain died at his home in Hampshire, England, October 2.

period;
f India
with a
ynopsis
philo-
Δ

nquiry.
face by
, 1919.

Lon-

Ethics.
O.

iversity

ted by
niver-

: The

ed and

ibner's

r, who
in the
After
ng his
ates in
untry.
most
a *The*

f Mr.
ied at